Banking Complexity in the Global Economy

Raoul Minetti[a], Giacomo Romanini[b], Oren Ziv*[a]

[a]Michigan State University

[b]Bank of Italy

Abstract

International lending flows are often intermediated through banking hubs and complex multi-national routing. We develop a dynamic stochastic general equilibrium model where global banks choose the path of direct or indirect lending through partner institutions in multiple countries. First, the model provides a lens to disentangle observed locational loan from unobserved ultimate lending: a bias originates both by attributing ultimate lending to banking hubs, and by missing ultimate lending that occurs indirectly via third countries. We next study the effects of global banking complexity. Indirect lending allows countries to bypass shocked lending routes via alternative countries; however, it dilutes their ability to diversify sources of funds after shocks. The quantitative analysis reveals that banking complexity can exacerbate credit and output instability when countries feature heterogeneous banking efficiency.

Keywords: Banks, Indirect Lending, Locational Loan Flows, Financial Integration

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1 Introduction

Recovering from the Global Financial Crisis retrenchment, banks' global cross-border claims now amount to more than $35 trillion (Claessens, 2017). An unknown but potentially substantial share of lending flows are intermediated between source and destination countries, through a ramified network of bank affiliates and subsidiaries (Allen, Gu, and Kowalewski, 2013) and complex financial routes through third countries (Coppola, Maggiori, Neiman, and Schreger, 2021). These indirect flows obscure the ultimate source of lending supply and demand from official statistics on bilateral flows of funds by conflating lending between countries with lending moving through them. Further, little is known about the role of these indirect banking linkages in the propagation of shocks, despite multinational banking groups their global operation being at the center of prudential policy to enhance financial stability (e.g. the European banking union debate).

We develop an N-country dynamic stochastic general equilibrium model where globally active banks choose the path of lending through an endogenously formed network of affiliates or partner institutions in multiple countries. Banking hubs arise endogenously in the financial intermediation network. The model provides a framework to reconcile observable international statistics that aggregate gross cross-border flows—which we refer to as *locational* flows—with notions of financial exposure. It generates a set of bilateral loan flows that conceptually matches the aggregate (observable) Bank of International Settlement (BIS) locational banking statistics (LBS), as distinct from the ultimate demand for and supply of liquidity from borrowers and savers—*ultimate* lending.[1] The model shows how conflating BIS LBS flows with ultimate loan demand and supply biases empirical results, and can even result in sign reversals. Moreover, it reveals that accounting for indirect lending is crucial for understanding the influence of global banking on the propagation of aggregate shocks.

In our economy, in each country banks produce loans using deposits and loan officer labor (e.g., as in Goodfriend and McCallum (2007)), and offer loan contracts internationally to firms. Extending loans internationally incurs costs in the form of regulatory, information, communication, and management frictions. Banks use

[1] Banking statistics that consolidate international banking groups also do not capture our notion of ultimate lending (Brei and von Peter, 2018). By comparing gross LBS flows (inflows plus outflows) to World Bank data on the size of domestic bank loan production, we isolate a set of countries that are most likely acting as global bank intermediation hubs.

their heterogeneous global networks to minimize these costs. They can either lend funds directly to firms in destination countries, or they can choose to lend indirectly through one or more third countries. Indirect lending can lower costs when, for example, subsidiaries or partners in third countries can more cheaply acquire information on and send liquidity to borrowers in destination countries. For instance, a US multinational bank may transfer funds to a subsidiary in the Netherlands whose loan officers can more effectively lend to firms in Belgium. Overall, global banks choose the cheapest option for their liquidity to reach final loan demand.

Following Allen and Arkolakis (2022), we model the choice of lending paths through intermediate countries by assuming that path costs are a product of all bilateral intermediation frictions—*edge costs*—incurred along the path as well as path-idiosyncratic costs. This allows us to aggregate the path choices of individual banks and to derive analytic expressions for the share of all bank lending from each origin country, to each destination country, using each intermediate country. The general equilibrium solution generates a gravity equation where the bilateral gravity friction is a non-geographic, endogenous outcome that corresponds to bilateral network proximity. The model yields a closed-form expression relating the ultimate origin-destination (supply-demand) lending, the cost structure of the network, and the locational flows of funds, equivalent of the BIS LBS. The model calibration yields estimates of international bilateral lending frictions and the elasticity of bilateral lending with respect to such frictions.[2] This elasticity governs the degree of *banking complexity* in our global economy: lower values of the elasticity imply a thicker tail of idiosyncratic draws, corresponding to a world where ramified banking networks are more active and, hence, indirect lending is more pervasive.

Our setting allows us to simulate the response of the global economy to shocks not only at nodes, corresponding to TFP or banking shocks in one country, but also to the edges connecting nodes, e.g. reflecting financial regulations or sanctions limiting cross-border lending. Such edge shocks are understudied but common: using the Barth, Caprio Jr, and Levine (2013) world regulation and supervision database, we document no less than 30 regulatory changes that potentially impacted cross-border lending between 1996 and 2012. In the case of node and edge shocks, we study how

[2]This elasticity, which we estimate as $\theta = 3.2$, can be thought of as corresponding to a trade elasticity in the context of global lending flows. Dordal i Carreras, Hoelzlein, and Orben (2023) estimate an elasticity of substitution for domestic interbank lending.

locational loan flows, ultimate origin-destination lending, as well as salient macroeconomic variables (investment and output) are affected, and we contrast their responses to those obtained in a network-free setting (i.e., where all loans flow directly to the destination country).

Using analytic results and simulations, we show how empirical specifications conflating the BIS LBS data with ultimate origin-destination lending bias results in two directions: first, by missing indirect loan flows between the countries, and second by counting funds not originated or destined for the countries as lending. First, we examine the propagation of a TFP shock hitting one country. Unaffected countries move lending away from the affected country and to other unaffected countries ("lending flight"), just as would be predicted in a model without indirect flows.[3] However, locational loan flows tell a different story: bilateral flows between unaffected countries *decrease*, picking up the reduction in lending to and from the shocked country that flows (indirectly) through unaffected countries. Next, we explore the impact of a shock to a single connection (edge) in the network. Here, the qualitative predictions of our model differ from a model without indirect lending. Further, locational flows can again move in opposition to the ultimate lending flows, incorrectly confirming the no-network predictions. These results underscore the need to disentangle locational flows from ultimate lending and may help explain third-country effects as empirically documented in Hale, Kapan, and Minoiu (2020) and Kalemli-Ozcan, Papaioannou, and Perri (2013), for example. Using case studies, we show how the model can help rationalize the behavior of loan flows observed in the occasion of two major shocks (the Estonian 1999 crisis and the banking liberalization of Poland in the late 1990s and 2000s).

Next, we operationalize our edge shocks to examine the influence of banking complexity on the international propagation of shocks. We consider complexity in the form of regulatory, policy, or technology changes that foster denser international networks supporting more indirect international lending.[4] The effects of banking complexity are multifaceted in our context: a complex banking network with more indirect lending paths can make countries more resilient to shocks to lending costs by

[3]Precisely, for any single *node* shock in our setting, an isomorphic single shock exists in a model with multinational banks but without indirect linkages. However, the same is not true for edge shocks.

[4]In our model, banking complexity corresponds to changes in the parameter governing the dispersion and relative importance of path productivity in loan intermediation. We also consider the effects of banking integration due to reductions in overall bilateral intermediation costs.

offering alternative paths through third countries. However, third countries' usage of indirect lending exposes their lending routes to shocks as well, inhibiting their ability to act as substitute sources of liquidity to shock-affected countries. Put differently, global banking complexity can increase the scope for diversification of lending *pathways*, but can reduce the scope for diversification of lending *sources*: as more banks rely on subsidiaries and partners in more countries for cross-border lending, an increase in intermediation costs in one location is more likely to impact the ability of third country banks to move liquidity internationally.

To quantitatively evaluate the relative strengths of the above mechanisms, we repeat the edge shock experiments in a series of increasingly complex environments. As the banking network becomes more complex (more indirect linkages), the benefit of improved path diversification dominates the reduced ability of the network to offer lending source diversification. On balance, in our baseline calibration, banking complexity moderates the drop in bilateral lending flows caused by an edge shock, thereby stabilizing investment and output in the shocked countries as well as overall global lending. Importantly, here too the mere observation of location loan flows would bias conclusions: increasing banking complexity leads to a larger response of locational lending, belying its stabilizing influence.

However, we also find that the relative strengths of source and path diversification—hence the overall effect of banking complexity—can differ when countries feature highly heterogeneous banking efficiency or when their banking productivity responds endogenously to network connectivity. Intuitively, when the banking sectors of third countries are highly efficient, the dilution of source diversification induced by a more complex banking network is particularly harmful. In such scenarios, banking complexity, i.e. stronger indirect links, can have a destabilizing effect, amplifying rather than mitigating shocks.

Our first contribution is to the literature on the geography of banking and the determinants and aggregate implications of international lending flows. The BIS LBS is the most extensive source of international banking statistics. This has prompted many studies on the macroeconomics of banking to use locational lending data. However, the proliferation of indirect lending, due to the expansion of large multinational banking conglomerates and global syndicated lending markets as well as the growing exploitation of offshore banking centers (Coppola, Maggiori, Neiman, and Schreger,

2021) has resulted in an increasing misalignment between these statistics and ultimate bilateral lending. This may lead to biased conclusions about the effects of global banking on the international transmission of shocks (see, e.g., Kalemli-Ozcan, Papaioannou, and Perri (2013) for more on this point). Our DSGE model of indirect lending provides a conceptual framework for the relationship between ultimate and locational flows. We analytically describe the nature of these biases and study quantitatively their direction and magnitude following shocks.[5]

Our second contribution is to the literature on the macroeconomic effects of global banking and banking integration. Kalemli-Ozcan, Papaioannou, and Perri (2013), Niepmann (2015) , Cao, Minetti, Olivero, and Romanini (2021), and Morelli, Ottonello, and Perez (2022) stress that larger multinational banking groups can increase the exposure of countries to shocks hitting a common multinational lender. However, financial integration can also allow for better diversification of funding sources in the aftermath of shocks hitting individual countries. We highlight the consequences of financial complexity: the greater availability of alternative indirect lending paths and the possible dilution of the ability of countries to serve as alternative sources of funding. Our analysis is in spirit related to Fillat, Garetto, Götz, and Smith (2018), who investigate how the branches and subsidiaries composition of multinational banks affects the international transmission of shocks.[6] Our emphasis is not on the structure of branches and subsidiaries, however, but on the distinction between direct and indirect bank lending. In this respect, we provide a novel characterization of global banking complexity, encompassing both the organizational and geographical complexity (Cetorelli and Goldberg, 2014): the elasticity of bilateral lending with respect to international lending frictions relates country-level determinants of diversification and productivity, including tax arbitrage and regulation, to bank-level path optimization and efficiency (Buch and Goldberg, 2022).

Finally, the paper broadly relates to the theoretical and empirical literature on financial and production networks. Studies explore the international transmission of the Lehman Brothers collapse via syndicated loans (De Haas and Van Horen, 2012),

[5]A related group of studies examine gravity in banking. Using BIS LBS data, Buch (2005) and Papaioannou (2009) find that distance is a relevant predictor of cross-border bank lending. See also Portes and Rey (2005) and Aviat and Coeurdacier (2007). However, Delatte, Capelle-blancard, and Bouvatier (2017) show that an empirical gravity equation for banking does a poor job of rationalizing the data.

[6]For empirical studies, see, e.g., Cetorelli and Goldberg (2012) and De Haas and Van Horen (2012).

monetary policy transmission via production networks, business cycle synchronization via trade linkages (Juvenal and Monteiro, 2017), or the role of trade credit in amplifying financial shocks (Altinoglu, 2021). We embed an endogenously formed banking network in a DSGE multi-country model with banks and study the aggregate consequences of banking complexity through this framework. In this dimension, the analysis also speaks to a set of papers that explore financial contagion (Allen and Gale (2000), Elliott, Golub, and Jackson (2014), Acemoglu, Ozdaglar, and Tahbaz-Salehi (2015)). We define and focus on a different aspect of integration related to internal networks of global banking conglomerates exploiting indirect lending. Our framework allows us to study edge shocks, scenarios related to disruptions to international financial linkages, besides country (node) shocks. To our knowledge this approach and its application to a general equilibrium dynamic setting are novel to the literature. This leads us to disentangle distinct mechanisms, path and source diversification, and the dispersion of indirect lending as driving forces of shock propagation.

The remainder of the paper is organized as follows. In Section 2 we set up the model and solve for agents' decisions. Section 3 studies the equilibrium. In Section 4, we present the model calibration and assess the biases resulting from conflating locational and ultimate loan flows. Section 5 examines the effects of global banking complexity. In Section 6 we consider the case of endogenous banking efficiency. Section 7 concludes. Technical derivations and proofs are relegated to the Online Appendices.

2 Model Setup

In this section, we present a discrete-time dynamic general equilibrium model with $\mathcal{N} > 2$ countries and endogenous international banking linkages. We specify agents' problems and study their decisions.

In each country $i \in \mathcal{N}$ there are four sectors: the household sector, the firm (goods production) sector, the capital production sector, and the banking sector (comprising banking consulting firms and banks). All agents are owned by households, who consume a single final good produced by competitive firms, and supply labor to the goods production and banking sectors. Labor and capital are immobile across countries. Firms hire labor from households, purchase physical capital from capital producers, and demand a diversified aggregate loan to finance their capital investment. Banking consulting firms acquire individual loan varieties $\omega \in \Omega$ in competitive international markets and produce the aggregate loan demanded by firms via CES bundling.[7] Banks produce loans using households' deposits and labor (e.g., as loan officers) and offer debt contracts to banking consulting firms internationally through cost-minimizing lending paths.

2.1 Households

Households in country i earn a wage rate $w^H_{i,t}$ on labor supplied to the goods production sector ($H_{i,t}$). They also earn a wage rate $w_{i,t}$ on labor supplied to the banking sector as loan production activity ($M_{i,t}$). Further, they earn a gross rate of return $(1+R^D_{i,t})$ on deposit holdings $D_{i,t}$. They use their funds for consumption $C_{i,t}$ and saving through deposit holdings, solving:

$$\begin{aligned}
\max_{\{C_{i,t},D_{i,t},H_{i,t},M_{i,t}\}_{t\geq 0}} \quad & \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\ln C_{i,t} - k_H \frac{H_{i,t}^{1+\epsilon}}{1+\epsilon} - k_M \frac{M_{i,t}^{1+\varphi}}{1+\varphi} \right) \\
s.t. \quad & C_{i,t} + D_{i,t} = (1+R^D_{i,t-1})D_{i,t-1} + w^H_{i,t}H_{i,t} + w_{i,t}M_{i,t} + \Pi_{i,t}, \qquad (1)
\end{aligned}$$

where ϵ is the inverse Frisch elasticity for labor supplied to the production of goods and φ is the inverse Frisch elasticity for labor supplied to banking activities. The parameters k_H and k_M govern the disutility from labor in the two sectors. The transfers $\Pi_{i,t}$ received by households comprise profits distributed by firms ($\Pi^F_{i,t}$), banks ($\Pi^B_{i,t}$) and capital producers ($\Pi^K_{i,t}$).

Households maximize their lifetime utility by choosing consumption $C_{i,t}$, deposits $D_{i,t}$, labor supply $H_{i,t}$ to firms, and labor supply $M_{i,t}$ to banks.[8] Henceforth, we denote the households' stochastic discount factor by $\Lambda_{i,t,t+j} = \beta^t \frac{u'_{C_{i,t+j}}}{u'_{C_{i,t}}}$.

2.2 Capital Producers and Firms

Capital producers Capital producers invest in $I_{i,t}$ units of capital goods at the cost of $I_{i,t}\left[1+f(I_{i,t}/I_{i,t-1})\right]$ units of consumption goods, where the continuous, convex

[7] The framework can capture the range of financial products that firms need from banks, e.g. credit lines and term financing, among others, or reflect sectoral specialization of banking institutions.

[8] Households' and firms' optimizing conditions are reported in the Appendix.

function $f(\cdot)$, with $f(1) = 0$, $f'(1) = 0$, captures the adjustment cost in the capital-producing technology. They choose the amount of new capital $I_{i,t}$ to maximize the present discounted value of lifetime profits:

$$\max_{I_{i,t}} \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \Lambda_{i,0,t} \left\{ P_{i,t}^K I_{i,t} - \left[1 + f\left(\frac{I_{i,t}}{I_{i,t-1}} \right) \right] I_{i,t} \right\}, \tag{2}$$

where $P_{i,t}^K$ denotes the price of capital. $\Lambda_{i,t,t+j} = \beta^t \frac{u'_{C_{i,t+j}}}{u'_{C_{i,t}}}$ is the stochastic discount factor, since households own capital producers and are the recipients of any profits $\Pi_{i,t}^K$. The first order condition

$$P_{i,t}^K = \left\{ 1 + f\left(\frac{I_{i,t}}{I_{i,t-1}} \right) + \frac{I_{i,t}}{I_{i,t-1}} f'\left(\frac{I_{i,t}}{I_{i,t-1}} \right) \right\} - \mathbb{E}_t \Lambda_{i,t,t+1} \left(\frac{I_{i,t+1}}{I_{i,t}} \right)^2 f'\left(\frac{I_{i,t+1}}{I_{i,t}} \right) \tag{3}$$

implies that the marginal cost of capital production is equal to the price of capital.

Firms The representative firm in country i uses labor $H_{i,t}$ and capital $K_{i,t-1}$ to produce final good $Y_{i,t}$ via an increasing, concave, and constant returns to scale technology. Firms must obtain loans to finance purchases of capital. They do so by demanding bundled loans $X_{i,t}$ from banking consulting firms.[9]

Firms maximize the discounted sum of dividends distributed to households, subject to the budget constraint (4), the technological constraint (5), the law of accumulation of the capital stock (6), and the financing constraint (7):

$$\max_{\{H_{i,t}, K_{i,t}, X_{i,t}, I_{i,t}\}} \mathbb{E}_0 \sum_{j=0}^{\infty} \Lambda_{i,t,t+j+1} \Pi_{i,t}^F$$

$$s.t. \quad \Pi_{i,t}^F + P_{i,t}^K I_{i,t} + (1 + R_{i,t-1}^X) X_{i,t-1} = Y_{i,t} + X_{i,t} - w_{i,t}^H H_{i,t}, \tag{4}$$

$$Y_{i,t}(K_{i,t-1}, H_{i,t}) = A_{i,t} K_{i,t-1}^{\alpha} H_{i,t}^{1-\alpha}, \tag{5}$$

$$K_{i,t} = (1 - \delta) K_{i,t-1} + I_{i,t}, \tag{6}$$

$$X_{i,t} = P_{i,t}^K I_{i,t}. \tag{7}$$

$R_{i,t}^X$ denotes the net interest rate on loan bundles, δ is the capital depreciation rate and $A_{i,t}$ captures the aggregate total factor productivity (TFP).

[9] This is similar to Craig and Ma (2020), where firms delegate their borrowing from lending banks to larger, diversified intermediary banks.

2.3 The Banking Sector

The banking sector comprises banking consulting firms and banks. Banks collect deposits from households domestically and lend to domestic and foreign consulting banks, which aggregate loans and extend them to firms. There are two facets of the bank lending technology. First, banks produce loans using deposits and labor. Second, loans can reach ultimate demand in a destination country directly (through subsidiaries in that country) or indirectly through lending paths, p, involving (subsidiaries in) third countries.

Consulting firms In each country, competitive consulting banks combine loans of type ω from the lowest-cost international suppliers to produce an aggregate non-traded loan $X_{j,t}$ using a CES function:

$$X_{j,t} = \left(\sum_i \int_{\omega \in \Omega} x_{ij,t}(\omega)^{\frac{\sigma-1}{\sigma}} \, d\omega \right)^{\frac{\sigma}{\sigma-1}}, \tag{8}$$

where $\sigma > 1$ is the constant elasticity of substitution across loan varieties.

Banks For each loan variety ω, there is a set of risk-neutral, competitive banks in each country i.[10] We refer to banks by their loan variety index ω. To maintain tractability, we posit that banks return dividends to households at the end of each period. This framework (combined with the i.i.d. assumption introduced below) is effectively isomorphic to a setting where banks exit in every period, as in Boissay, Collard, and Smets (2016).[11] We accordingly drop the time subscript from banks' problem.

[10] ω banks could be differentiated in terms of maturity, industry, or other contract characteristics. As in Eaton and Kortum (2002), ω banks within countries are homogeneous, including their idiosyncratic path choices. This can be relaxed by considering a large-N number of banks of each ω type with heterogeneous networks available to them, i.e. heterogeneous path choices. The banking sector could be alternatively modeled via monopolistic competition, as in Gerali, Neri, Sessa, and Signoretti (2010). We provide a Melitz (2003) version of the banking partial equilibrium in a supplement.

[11] In our model, their setting would imply that banks are born and collect deposits in $t-1$, hire workers in period t, and die at the end of period t, so that deposit rates, wages, and profits are payed to households in period t. Differently from Boissay, Collard, and Smets (2016) (BCS), the Eaton and Kortum (2002) structure implies that shocks are realized before banks choose inputs. This implies that individual banks do not face input mismatches, neither for liquidity (as in BCS) nor for labor. The model could also be augmented with an explicit interbank market, as shown in a supplement.

Loan production Building on Goodfriend and McCallum (2007), banks produce loans by combining labor $M_i(\omega)$ and deposits $D_i(\omega)$ via a Leontief technology.[12] Labor tasks in banks can comprise deposit collection, information production, and headquarter services. Banks maximize dividends taking as given the banking sector wage w_i and the net interest rate on deposits R_i^D. A loan $x_{ij}(\omega)$ destined for a firm in country j entails a route-dependent intermediation cost $\tau_{ij}(\omega,p) \geq 1$, modeled as an interest rate markup.[13]

Since the choice of inputs does not affect the lending path optimality, and viceversa, we present the two subproblems (loan production and loan path selection) sequentially. Here, we describe banks' input choice problem in loan production given a lending path p. Substituting the usual asset and liabilities balance, a bank's problem reduces to maximizing its net cash flow subject to the loan production function:

$$\max_{M_i(\omega),D_i(\omega)} \sum_j \frac{r_{ij}(\omega)}{\tau_{ij}(\omega,p)} x_{ij}(\omega) - w_i M_i(\omega) - R_i^D D_i(\omega) \tag{9}$$

$$s.t. \quad \sum_j x_{ij}(\omega) = \min\{z_i M_i(\omega), D_i(\omega)\} \tag{10}$$

where $r_{ij}(\omega)$ is the net interest rate in j, and z_i is a country-specific labor productivity in the banking sector.[14] Competition implies that banks from i charge a net interest rate in j equal to their marginal cost:

$$r_{ij}(\omega,p) = c_i \tau_{ij}(\omega,p) \tag{11}$$

where the unit loan production cost, c_i, net of the bilateral intermediation cost, satisfies:[15]

$$c_i = \frac{w_i}{z_i} + R_i^D. \tag{12}$$

[12]Our model can be related to the two-tier Gerali, Neri, Sessa, and Signoretti (2010) banking structure, since the Leontief production function makes the two inputs (deposits and loan officers) complements and both necessary for the production of loans. A Cobb-Douglas loan production technology as in Goodfriend and McCallum (2007) would imply $c_i = a(w_i/z_i)^{\zeta}\left(R_i^D\right)^{1-\zeta}$ with a being a constant.

[13]This explicitly incorporates the cost into banks' optimization problem, without resorting to the notion of iceberg losses as in standard gravity models.

[14]In our main exposition we treat z_i as exogenous. In Section 6 we relax the exogeneity assumption by making it network-dependent.

[15]All banks have the same technology, hence $c_i(\omega) = c_i$.

The ex-ante bilateral intermediation friction $\tau_{ij}(\omega, p)$ is both bank ω and path p specific. In what follows, we study banks' lending path decisions.

Loan path selection The decision facing banks of how to move funds internationally closely follows Allen and Arkolakis (2022). A path, or route, is a graph $p \in P$ which consists of an ordered sequence of nodes (countries) $\mathcal{N}(p)$ of length K_p and a set of edges $E(p)$ that connect nodes, following a sequence beginning from an "origin" node i and ending with a "destination" node j, i.e. $\mathcal{N}(p) = \{i, k_1, \ldots, j\}$, and $E(p) = \{\{i, k_1\}, \{k_1, k_2\}, \ldots, \{k_{N-1}, j\}\}$.

Banks have the option to move funds along any edge, from any country (node) k to any country (node) l. Moving liquidity along any path p is costly and incurs intermediation frictions. These frictions represent the cost of monitoring foreign subsidiary intermediaries, which entails informational frictions, frictions for using internal capital markets, and loan officer training efforts. For example, a US multinational bank interested in extending a loan to a Polish firm might choose to extend the loan directly through a (recently established) subsidiary in Poland, or instead choose to first transfer liquidity to a (well-established) subsidiary in the Czech Republic, which could then arrange a loan trough the Polish correspondent of the banking conglomerate. Monitoring and managing the relationship with the Czech subsidiary entails costs, and in turn the Czech subsidiary will face costs for monitoring the activity of the correspondent in Poland. Still, this might me cheaper than dealing with the Polish subsidiary directly, if the established Czech subsidiary can easily communicate with the US headquarters and is particularly familiar with the Polish market. In Appendix A, we microfound the "reduced form" intermediation costs of the model along these lines, as intermediation and monitoring costs of intermediary agents at each node.[16]

To minimize a loan's intermediation cost $\tau_{ij}(\omega, p)$, a bank chooses the lowest-cost path through which to send the loan. Banks in i can extend a loan to j through the direct path $p_{direct} = \{i, j\}$ or indirectly through one or more intermediary locations k comprising a path of length greater than one. Specifically, a component of the intermediation costs, $e_{kl} > 1$, is shared by all banks wishing to move loans along a given edge. This corresponds to challenges facing all banking groups intermediating through the edge, and may include common regulatory, language or cultural barriers

[16] In the main text, intermediation wages are paid in the originating country. In Appendix A, we explore extensions where intermediation requires that labor wages are paid along the path as well, similar to Antràs and De Gortari (2020).

or other common, uniform informational frictions. Other frictions are instead specific to a bank's subsidiaries and a given arrangement. For example, such specific costs may capture the difficulty a given set of loan officers and report structures face in managing and monitoring loans of a specific type or industry.

A bank ω's cost $\tau_{ij}(p,\omega)$ of sending funds from i to j through path p can therefore be decomposed into the path-specific series of edge costs that are shared by all banks and the bank, path-specific cost:

$$\tau_{ij}(p,\omega) \equiv \frac{\tilde{\tau}_{ij}(p)}{\xi_{ij}(p,\omega)}. \tag{13}$$

$\tilde{\tau}_{ij}(p) \equiv \prod_{k=1}^{K_p} e_{k-1,k}$ is the (deterministic, shared) portion of costs related to bilateral liquidity transfers and $\xi_{ij}(p,\omega)$ is the intermediation cost of moving a loan of a given type ω from i to j that is specific to the organizational structure of affiliates associated with p.[17] We further assume $\xi_{ij}(p,\omega)$ is an i.i.d. realization of draws from a Fréchet distribution with shape parameter $\theta > \max\{1, \sigma - 1\}$: $F(\xi_{ij}(p,\omega)) = \exp\left\{-\xi^{-\theta}\right\}$.[18] The shape parameter θ governs the degree of heterogeneity in the productivity of affiliate groups and organizational structures. This can be thought of as a technological parameter, encompassing anything amplifying differences in loan officer group productivity, e.g. information communication technology or the regulatory environment. When θ is small, the bank-specific portion of intermediation costs dominates the path choice.[19]

ω-banks at i offering loans to j choose the least cost path to send loans, such that:

$$\tau_{ij}(\omega) \equiv \min_{p \in P} \tau_{ij}(p,\omega). \tag{14}$$

3 Equilibrium

We first derive the partial equilibrium probabilities of loan and route choices, then distinct equilibrium gravity equations for locational and ultimate loan flows, and fi-

[17] An alternative would be to model the edge frictions as additive. We prefer the multiplicative form which is consistent with intermediary interest markups.

[18] The i.i.d. assumption allows to maintain tractability and obtain analytical results.

[19] This framework implies that each bank has a full network of intermediaries. An isomorphic alternative is to model each bank as having an exogenous set of intermediaries (and thus path choices) at a subset of countries. Specifically, for a given set of idiosyncratic draws, the observed paths selected in equilibrium are identical.

nally close the model.

3.1 Path Probabilities

After banks identify least-cost paths, consulting firms in j choose the lowest-cost supplier of loan ω from all countries $i \in \mathcal{N}$. Using the standard Eaton and Kortum (2002) method, summing across loan types for each path, we obtain the following:

Lemma 1 (Gravity probability)**.** *The probability that borrowers in country j choose to obtain a loan from country i through a path p is:*

$$\lambda_{ijp\omega} = \frac{\left[c_i \tilde{\tau}_{ij}(p)\right]^{-\theta}}{\sum_l c_l^{-\theta} \sum_{p \in G} \left[\tilde{\tau}_{lj}(p)\right]^{-\theta}}. \tag{15}$$

Proof. See Appendix B.2.

Similar to Eaton and Kortum (2002), across all loan types, this is also the share of loans extended in country j that come from i and take path p, λ_{ijp}. Overall, countries with lower loan production costs and with paths incurring lower bilateral intermediation costs will account for a larger fraction of the loans extended in country j.

3.2 Network Costs

We can define the term $\sum_{p \in G_{ij}} \tilde{\tau}_{ij}(p)^{-\theta}$ in the denominator of (15) as the *bilateral network cost*:

$$\tau_{ij} \equiv \sum_{p \in G_{ij}} \tilde{\tau}_{ij}(p)^{-\theta} \propto E(\tau_{ij}(p)). \tag{16}$$

τ_{ij} is proportional to the average or expected bilateral cost, summing over all the possible paths' costs conditional on selection. It is a single measure that summarizes direct and indirect frictions between country i and country j. Reordering paths by length K, we can consider this sum as capturing K-th order connections between i and j and, as such, it is inversely related to the (measure of the) network proximity between i and j:

Lemma 2 (Network Costs)**.** *Let A be the inverse cost matrix where the ijth element of A*

is e_{ij}, and let b_{ij} denote an element of the matrix $B \equiv (\mathbf{I} - A)^{-1}$. For a constant γ

$$\tau_{ij} = \gamma b_{ij}^{-1/\theta}. \tag{17}$$

Proof. See Appendix B.3.

In the remaining of the paper, we sometimes prefer working directly with the matrices A and B rather than with the edge costs e or the network costs τ. With a slight abuse of interpretation, we call these edge and network probabilities, respectively.

3.3 Ultimate and Locational Gravity

We now describe how the above network probabilities (and hence, the network costs) give rise to aggregate variables: ultimate (X_{ij}) and locational (Ξ_{ij}) bilateral loan flows, and the aggregate interest rate that firms face in a given country, R_j^X.

We first derive a closed-form expression for the composite loan interest rate $R_{j,t}^X$:

$$R_j^X = \vartheta \left(\sum_i \left(c_i \tau_{ij} \right)^{-\theta} \right)^{-\frac{1}{\theta}} \tag{18}$$

where $\vartheta = \Gamma\left(\frac{\theta+1-\sigma}{\theta}\right)^{\frac{1}{1-\sigma}}$ is a constant.[20] The derivation is in Appendix B.5.

Ultimate gravity Starting from lemma 1, summing across routes and using the network probabilities, we obtain the share of total loans extended in country j obtained from country i:

$$\lambda_{ij} = \frac{\left(c_i \tau_{ij} \right)^{-\theta}}{\sum_l \left(c_l \tau_{lj} \right)^{-\theta}} = \frac{\left(c_i \tau_{ij} \right)^{-\theta}}{\vartheta^\theta R_j^{-\theta}}. \tag{19}$$

Defining the aggregate loans in country j obtained from i as $X_{ij} \equiv \lambda_{ij} X_j$, we recover a network gravity equation:

$$X_{ij} = \vartheta^{-\theta} R_j^\theta X_j c_i^{-\theta} \tau_{ij}^{-\theta}. \tag{20}$$

[20] Note that in a one-country setup the aggregate interest rate would be equal - simplifying the weights - to $R_i^X = w_i + R_i^D$, and we would obtain a simple expression similar to the interest rate spreads in Goodfriend and McCallum (2007) or Gerali, Neri, Sessa, and Signoretti (2010).

This gravity equation relates bilateral ultimate lending to loan demand in j, $R_j^{\theta} X_j$, loan production efficiency in i, $c_i^{-\theta}$, and a bilateral friction, $\tau_{ij}^{-\theta}$. However, recalling that $\tau_{ij}^{-\theta} = \left(\sum_{p \in G_{ij}} \prod_{k=1}^{K_p} e_{k-1,k}(p)^{-\theta} \right)$, rather than a primitive, this bilateral friction is the average cost of lending across paths of different length through the banking network. It accounts for the effect of the complexity of the entire banking network on ultimate bilateral lending, including the possibility of different path lengths and third country effects.

Locational gravity Bank loans can take a direct path from country i to country j, as well as an indirect path, through a series of countries $k, l, \ldots, K$. In this section, we derive a *locational* gravity equation, i.e. the amount of loans that go through an edge e_{kl}, without necessarily originating at k nor stopping at l. Let $\psi_{kl|ij}$ denote the conditional probability of a bank ω serving country j from county i, and going through countries k and l. We can prove the following:

Lemma 3. *The probability of any loan product ω going through a kl edge, conditional on being originated in i and ultimately extended in j, is*

$$\psi_{kl|ij} = \frac{b_{ik} a_{kl} b_{lj}}{b_{ij}} = \left(\gamma \frac{\tau_{ij}}{\tau_{ik} e_{kl} \tau_{lj}} \right)^{\theta}. \tag{21}$$

Proof. *See Appendix B.4.*

The first expression for the conditional probability $\psi_{kl|ij}$ in (21) can be visualized in Figure 1. The link probability, a_{kl}, is the probability of going from k to l, conditional on the rest of the banking network. The network probabilities b, instead, capture all the possible paths from origin country i to the intermediate step k (upstream), and all the possible paths from the intermediate step l to the destination country j (downstream); where upstream and downstream are relative to the intermediate step kl. The second expression for $\psi_{kl|ij}$ in (21), written in terms of costs, relates the probability of using kl to the cost e_{kl} and the average network cost of sending loans from i to k and from l to j.

$\psi_{kl|ij}$ is a microfounded, probabilistic version of the edge betweeness centrality formula, such that the importance of an edge depends on the amount of shortest lending paths that go through it.[21]

[21] To express node betweeness centrality, we can present a triangular ikj version of the same, which

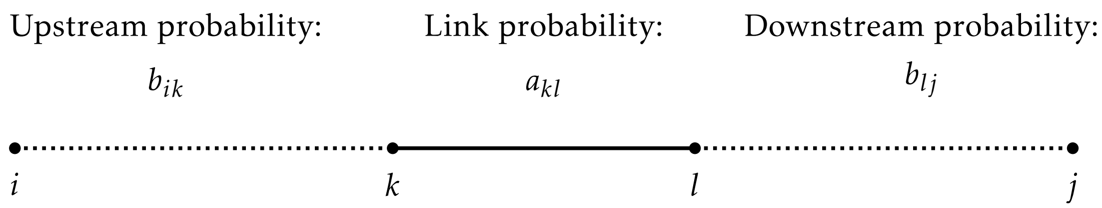


Figure 1: Locational Gravity: Probability Interpretation

Note: The figure draws a schematic representation of indirect lending. i k l and j are nodes or countries. The dotted lines between i and k and l and j represent the sets of possible paths between the respective nodes. The solid line represents the edge between k and l.

We can now obtain the total locational loan flows between k and l, Ξ_{kl}, summing over all the origin i and destination j countries:

$$\Xi_{kl} = \sum_i \sum_j X_{ij} \psi_{kl|ij} = \sum_i \sum_j X_{ij} \frac{b_{ik} a_{kl} b_{lj}}{b_{ij}}. \tag{22}$$

This locational loan volume includes loan flows bound for l and those continuing onward to other destination countries. Locational loan flows Ξ are a function of loan intermediation costs and ultimate loan flows X only. The effects on loan flows of country-specific features such as loan production efficiency and equilibrium loan rates, as well as shocks to these, are subsumed by X, and affect locational flows proportionately on all routes according to $\psi_{kl|ij}$.[22]

Is the United Kingdom the same as the Cayman Islands? The equilibrium delivers a set of both ultimate and locational bilateral loan flows. A country with a solid presence of internationally active banks, e.g. Italy, would be characterized by sizeable

is more intuitive: Ξ_{ikj} is the probability of a loan going from country origin i to destination country j through a third country k,

$$\Xi_{ikj} = \sum_l \psi_{kl|ij} X_{ij} = \sum_l \psi_{kl|ij} \lambda_{ij} X_j = \psi_{k|ij} \lambda_{ij} X_j,$$

where the last expression uses the definition of the bilateral loan flows, as the product of loan demand and the share λ. This is related to equation 7 in Arkolakis, Ramondo, Rodríguez-Clare, and Yeaple (2018) model of multinational production.

[22]Since, following Allen and Arkolakis (2022), there is a unique set of ultimate loan flows X consistent with observed country characteristics, market clearing conditions, and banks' optimization, there exists a unique matrix Ξ of locational loan flows.

ultimate loan flows $X_{\mathrm{ITA},j}$. Countries with a developed banking sector that also act as banking hubs, e.g. Great Britain, would originate *and* intermediate loans, hence exhibiting high ultimate flows $X_{\mathrm{UK},j}$ *and* locational flows, both outward $\Xi_{\mathrm{UK},l}$ and inward $\Xi_{k,\mathrm{UK}}$. Offshore banking countries, instead, would be characterized by lower ultimate flows, both inward and outward, while exhibiting high locational flows: they generate a small volume of loans, but they facilitate the intermediation of loans through their jurisdiction.

The comparison between locational loan flows and loans originated in a country allows to distinguish different scenarios. For example, one could consider the statistic $(\Xi_{ik} + \Xi_{kj})/\sum_j X_{kj}$, which measures the total flow of locational loans in and out of country k relative to the total loans originated in country k (regardless of destination). Holding fixed the ratio of domestic to international loans, two countries (nodes), such as the United Kingdom and the Cayman islands, could both have large locational loan flows going through them (i.e., a high value of $(\Xi_{ik} + \Xi_{kj})$) but exhibit a very different value of this statistic. The Caymans would have a high value of the above ratio, as they intermediate a large volume of international loans while producing very few; Great Britain would feature a relatively lower value of the ratio as it produces a substantial amount of loans, besides acting as an intermediary node in the global banking network.

To probe these points in the data, we gathered information on locational loan inflows and outflows for all BIS reporting countries from the BIS Locational Banking Statistics, together with data from the World Bank Financial Development Database on total domestic deposits in the same countries. We considered two years: 2001, at the onset of the process of global expansion of multinational banking groups, and 2017, after the recovery of global lending flows from the Great Financial Crisis. Figure 2 plots the gross locational loan flows (inflows plus outflows) of the countries against the volume of their domestic deposits (both expressed in log million dollars). The blue and red circles refer to 2001 and 2017, respectively, with the size of the circles reflecting the size of the economies (GDP). Fitted lines are also shown. The figure helps to distinguish between banking hub countries, plotted above the fitted line, and non-banking hub countries (e.g., Chile or Canada), which fall below it. Further, pure banking hubs with modest own loan production (e.g., the Bahamas and Cyprus) are plotted in the upper-left region while banking hubs also featuring large own loan

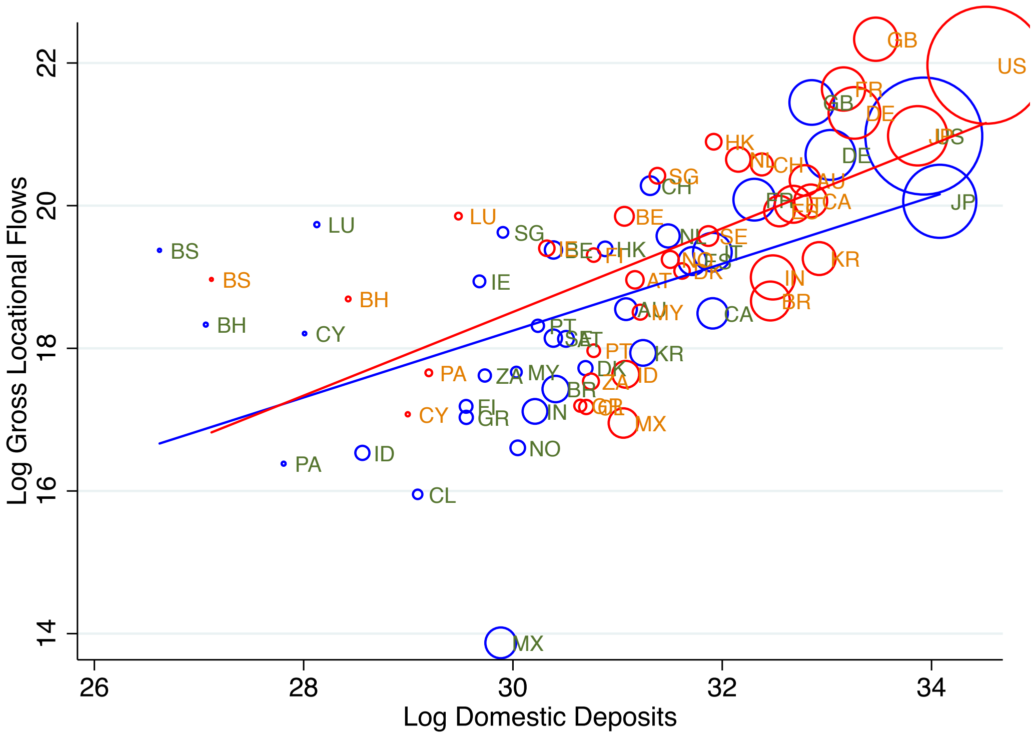


Figure 2: Gross Locational Loan Flows vs Deposits

Note: The figure plots the log of the sum of gross incoming and outgoing locational flows (BIS LBS) (y axis) against the log domestic deposits with circle sizes denoting national GDP (World Bank Global Financial Development Database) and fit lines in 2017 and 2001. Red circles and fit line use 2017Q4 data. Blue circles and fit line represent 2001Q4 data. ISO country codes are plotted next to circles in green and orange, respectively. Where, for some countries, BIS or World Bank data was missing for 2001Q4, we take data from the earliest possible date after 2001Q4.

production (e.g., Great Britain) are plotted in the upper-right region. The figure also hints to the increase in the complexity of global banking from 2001 to 2017: the upward shift of the fitted line suggests an overall increase in gross locational loan flows for any size of domestic bank loan production. More surprisingly (but in line with policy efforts to reduce the relevance of bank tax heavens), the increase in the slope of the fitted line from 2001 to 2017 suggests that such complexification was driven more by banking hubs with sizeable domestic bank loan production than by hubs with modest loan production.

3.4 Closing the Model

For each country i, we obtain banks' aggregate demand for labor and deposits by aggregating across the individual loan varieties:

$$M_{i,t} = \int_{\Omega} M_{i,t}(\omega) d\omega, \qquad D_{i,t} = \int_{\Omega} D_{i,t}(\omega) d\omega. \tag{23}$$

Banks' aggregate demand for labor and deposits is related to the total loans produced in the country (regardless of destination), $X_{ij,t}$:

$$M_{i,t} = \sum_{j} X_{ij,t}/z_i. \qquad D_{i,t} = \sum_{j} X_{ij,t}. \tag{24}$$

A competitive equilibrium is defined in the usual way: all agents optimize taking prices and interest rates as given, and the markets for both types of labor, capital, deposits, aggregate and individual loans, and final goods clear.

4 Model Analysis

We study the effects of shocks through the model and simulations. The sources of disturbance consist of shocks to nodes, in the form of TFP shocks, and shocks to edges, in the form of changes to bilateral intermediation costs e_{ij} between country pairs.[23] We examine the responses of locational and ultimate loan flows as well as the responses of investment, labor, and output.

[23]We consider temporary AR(1) shocks, but in Appendix C also extend the analysis to permanent shocks.

In this section, we investigate the effects of node and edge shocks on both ultimate and locational flows. We derive then illustrate the bias that arises from observing locational loan flows in place of ultimate lending. Locational loan flows can distort the measure — or even reverse the sign — of a shock's impact on ultimate lending. In the subsequent section, we investigate the implications of indirect lending for the global propagation of shocks and for aggregate stability.

4.1 Calibration

In what follows, when simulating the dynamic behavior of our model economy following shocks, we consider a framework with $\mathcal{N} = 3$ countries. The results are robust to scenarios with $\mathcal{N} = 5$ countries.[24]

The model is calibrated to quarterly frequency and solved numerically by locally approximating around the non-stochastic steady state. Table 1 presents calibrated values for the model parameters. Agnostically, and in line with a broad DSGE literature on banking in multi-country settings, we calibrate parameters symmetrically across countries, and make reference to advanced economies such as the euro area countries, the United States, or the United Kingdom (Cao, Minetti, Olivero, and Romanini, 2021, Kalemli-Ozcan, Papaioannou, and Perri, 2013). We explore asymmetric calibrations of banking sector productivity and edge costs in sensitivity analyses. These experiments may capture alternative specific scenarios such as banking integration between advanced and emerging economies. In principle such asymmetric calibrations could also be extended to case studies of integration between sharply different countries, targeting specific sizes and flows.

In total, there are thirteen parameters to be calibrated. Five parameters refer to the household sector: the discount factor β, the inverse Frish elasticities for labor supplied to the firm sector ϵ and to the banking sector φ, and the labor disutility parameters in the two sectors, k^H and k^M. Three parameters pertain to the firm sector: the capital share of output α, the depreciation rate δ, and the parameter governing the elasticity of the capital supply η. Five parameters refer to the banking sector: the elasticity of substitution σ across loan varieties, the loan officers' productivity z, the Frechet shape parameter θ, the cross-border edge cost e_{ij}, and the internal

[24] Closed-form expressions are computationally infeasible for $\mathcal{N} > 5$.

intermediation cost e_{ii}. We externally calibrate seven parameters (β, ϵ, φ, α, δ, η, σ), which are fairly standard in the literature. We internally calibrate six parameters (k^H, k^M, z, θ, e_{ij}, e_{ii}) by targeting data moments.

Consider first the externally calibrated parameters. In the household sector, the labor supply elasticity is set to $\epsilon = \varphi = 0.25$ in both the final goods and the banking sectors, as in Gertler and Karadi (2011). The discount factor is calibrated to $\beta = 0.9975$, implying a yearly steady state deposit rate (R^D) of 1%, as in Goodfriend and McCallum (2007). In the firm sector, the depreciation rate of capital is set to $\delta = 0.025$ implying an annual depreciation rate of 10%, in line with a broad class of models. We set the share of capital $\alpha = 0.3$, implying a wage bill of two thirds of total output. In the capital producing sector, we specify the investment adjustment function as $f(I) = -\eta/2\,(I_t/I_{t-1} - 1)^2$, with $\eta = 1.728$, as in Gertler, Kiyotaki, and Queralto (2012). In the banking sector, we set the elasticity of substitution across loan product varieties at $\sigma = 1.471$ as in Gerali, Neri, Sessa, and Signoretti (2010). This implies a limited degree of substitutability across loan varieties and, hence, across banks, in line with the observation that relational lending is pervasive in credit markets and firms cannot easily switch among lending banks (see, e.g., Chodorow-Reich (2014)).

Turning to the six internally calibrated parameters (k^H, k^M, z, θ, e_{ij}, e_{ii}), we pick them so as to match the following targets: total hours worked ($M + H$) relative to GDP, the ratio of the wage in the banking sector relative to the wage in the goods producing sector, the ratio of non-interest operating (labor) expenses in the banking sector over total (interest and non-interest) bank expenses, the ratio of cross-border lending to domestic lending, the spread between the loan and the deposit rates, and the wedge between the financing cost of loan products extended internationally and domestically. We pick the labor disutility parameters (k_i^H and k_i^M) and the loan officers' productivity z_i so that, in conjunction with the other parameters, we match total hours worked equal to 1/3 of GDP, a ratio of the wage in the banking sector relative to the wage in the goods producing sector of four thirds, and a ratio of bank non-interest operating expenses over total bank expenses of 93.0%. In the US data the hourly wage is around \$40 in the financial sector and \$30 in manufacturing; similar ratios can be observed in the euro area (BLS, 2023, Eurostat, 2021). The targeted ratio of non-interest expenses over total bank expenses is drawn from the US FDIC Call reports.

Table 1: Calibration of Selected Parameters

Description	Symbol	Value
Preferences		
Household discount factor	β	0.960
Inverse Frisch elasticity for H	ϵ	0.250
Inverse Frisch elasticity for M	φ	0.250
Firms		
Capital share of output	α	0.330
Capital depreciation	δ	0.025
Inverse elasticity of net investment to the price of capital	η	1.728
Banks		
Loans elasticity of substitution	σ	1.471
Fréchet shape parameter	θ	3.262
Cross-border edge cost	e_{ij}	1.98
Internal intermediation cost	e_{ii}	1.23

Note: This table presents the calibrated values of selected parameters of the model.

We calibrate the Fréchet parameter to $\theta = 3.262$, the cross-border edge cost to $e_{ij} = 1.98$ and the internal intermediation cost to $e_{ii} = 1.23$ so that, in conjunction with the other parameters, we match a ratio of cross-border lending to domestic lending of 1 to 4, a quarterly spread between the aggregate loan rate and the deposit rate of 150 basis points (6% annually), and a quarterly wedge between the financing cost of loan products extended internationally and domestically of 40 basis points (1.6% annually). The targeted ratio between cross-border and domestic lending of 1 to 4 is in line with the ratio (25%) reported in BIS data on foreign versus domestic loan claims for the set of BIS reporting countries.[25] The targeted annual spread between the loan and deposit rates of 6% is in line with observations for global bank loans over recent decades. For the 1995-2013 period, Lee, Liu, and Stebunovs (2017) estimate an average value of the spread between the loan rate on global loans and the LIBOR rate around 150 basis points annually (excluding the Great Recession spike).[26] Moreover, in the same period the gap between the LIBOR and the US deposit rate was about 450 points annually. Finally, the studies above document that the foreign origin of lenders can explain between one fourth and one third of the interest rate charged on loans.

We verified that the calibrated values of the Fréchet parameter and of the edge cost parameters e_{ij} and e_{ii} satisfy the condition that the spectral radius of the matrix

[25] This is also in the ballpark of the average for major euro area countries.

[26] Gao and Jang (2021) and Liu and Pogach (2016) document a similar spread.

A (defined in lemma 2) is less than 1.[27] In Sections 5.1-5.2, we will investigate the role of the Fréchet parameter θ and the sensitivity of the results to its value. In Section 5.3, we will discuss the role of different network topologies (different, possibly asymmetric values of e_{ij} across country pairs).

4.2 Confounding Ultimate and Locational Loan Flows

We investigate the importance of distinguishing between ultimate and locational loan flows, first in the context of a node shock and then of an edge shock. The empirical literature has often preferred locational data for two reasons: cross-sectional and time-span availability, and the argument that locational loan flows better capture bilateral exposures, contagion, or synchronization. On the other hand, empirical studies (e.g., Hale, Kapan, and Minoiu, 2020) detect major puzzles when using such locational loan flows, as discussed below.

Recalling equation (22), we can write the bias when using locational loan flows Ξ_{kl} in place of ultimate loan flows X_{kl}:

$$\Xi_{kl} - X_{kl} = \underbrace{\sum_{\{i,j\}\setminus\{k,l\}} X_{ij}\psi_{kl|ij}}_{\substack{\text{locational flows associated} \\ \text{with indirect lending}}} - \underbrace{X_{kl}(1-\psi_{kl|kl})}_{\substack{\text{ultimate flows not} \\ \text{transmitted through } kl}} \quad . \tag{25}$$

The first term is the total amount of loans from all sources i to all destinations j, through kl. It represents a non-negative error that comes from attributing locational loan flows through kl to ultimate kl lending. This would be large, for example, in the case of tax haven countries with low loan origination. The second term represents a negative error coming from ultimate flows that originate in k for destination l that circumvent the edge kl by using other network nodes. Overall, the more a node k is used as a banking hub for l, the more empirical work using Ξ_{kl} will overstate lending flows from k to l; the more lending from k to l flows through alternative banking hubs, the more locational flows will understate lending. The sign of the net error for each edge kl is ultimately a function of the network structure. The direction of the bias arising from these errors will be a function of the correlation between the net

[27] This is in order to apply the geometric sum that leads to matrix B. A sufficient condition for the spectral radius being less than 1 is $\sum_j e_{ij}^{-\theta} < 1$ for all i, which is satisfied in the calibration.

error and any variable of interest.

4.2.1 Node (TFP) shocks

We first explore the bias from equation (25) when a country is hit by a negative TFP shock. Changes to bilateral loan flows happen via general equilibrium effects which induce changes in interest rates or domestic wages which affect ultimate and *then* locational loan flows. However, node shocks do not affect the network structure and their effects are isomorphic to the effects in a model without indirect lending. We therefore do not expect qualitative differences between the responses of ultimate lending or real sector variables in the two models.[28] On the other hand, the effects on locational flows will be different. In Section 6, we develop an extension in which the effects of a node shock on ultimate loan flows also diverge between the two models.

Proposition 1 (Node Shock Impact on Ultimate and Locational Flows)**.** *A TFP shock in country k results in the following change in ultimate and locational loan flows:*

$$\frac{\partial X_{ij}}{\partial TFP_k} = \frac{\partial X_j}{\partial TFP_k}\lambda_{ij} + X_j \frac{\partial \lambda_{ij}}{\partial TFP_k}, \qquad \frac{\partial \Xi_{k',l'}}{\partial TFP_k} = \sum_{i,j} \frac{\partial X_{ij}}{\partial TFP_k}\psi_{k'l'|ij}. \tag{26}$$

Hence, the bias from observing locational loan flows is:

$$\sum_{\{i,j\}\backslash\{k,l\}} \left(\underbrace{\frac{\partial X_j}{\partial TFP_{k'}}\lambda_{ij}}_{\substack{\text{GE effects on}\\ \text{indirect flows}}} + \underbrace{X_j \frac{\partial \lambda_{ij}}{\partial TFP_{k'}}}_{\substack{\text{Indirect}\\ \text{substitution lending}}} \right) \psi_{kl|ij} - \left(\underbrace{\frac{\partial X_l}{\partial TFP_{k'}}\lambda_{kl} + X_l \frac{\partial \lambda_{kl}}{\partial TFP_{k'}}}_{\substack{\text{Effect on flows not}\\ \text{transmitted through } kl}} \right) (1 - \psi_{kl|kl}). \tag{27}$$

The expression in (26) for the impact of a node (TFP) shock on ultimate loan flows $\frac{\partial X_{ij}}{\partial TFP_k}$ would look the same in a standard gravity model without indirect lending. However, this is due to the fact that in our model the complexity of the network is captured via expected costs τ_{ij} which, in turn, affect the loan shares λ_{ij}. Equation (26) further shows that the impact on the locational loan flows between k' and l' is the weighted average effect of the impact on the ultimate flows, with weights being the network relevance of the edge $k'l'$ in the paths starting in all origins i to all destina-

[28] Indeed, the effects would be quantitatively identical in a model without indirect lending in which the bilateral frictions are exactly equal to the bilateral network costs in our model.

tions j. Finally, equation (27) shows how observed locational loan flows belie the true bilateral effects of the shock. When the shock incentivizes to move loans through a link or substituted lending to move more funds indirectly through that link, the bias tends to be positive, while the true bilateral effects are understated to the extent that funding avoids flowing through that bilateral link.[29]

Illustrating the proposition, Figure 3 shows the impulse responses for all loan flows to a 1 percent negative TFP shock in country 1.[30] The solid blue lines correspond to ultimate loan flows determined by equation (20), while the blue dashed-dotted lines correspond to locational loan flows determined by equation (22). In the figure we compare the responses with the responses of loan flows in an alternative gravity model, i.e. an economy with the same bilateral frictions but without network paths and indirect lending (black dashed lines).[31]

As expected, both our model and the comparison economy agree on the direction of the shock's effect on the ultimate loan flows. As standard in the IRBC literature, a negative TFP shock in country 1 reduces investment in that country, and hence the amount of its domestic credit, X_{11}. Absent other frictions, it also leads to a credit flight, that is, a decrease in X_{21} and X_{31}.[32]

The nine panels also demonstrate the bias in equation (27). Most remarkably, the direction of response of ultimate and locational loan flows between non-shocked countries (2 and 3) are reversed. Both models agree that ultimate loan flows increase. Country 2 diverts lending away from now-less-productive firms in 1 towards firms in country 3. But, because part of the ultimate loans to and from 1 flow indirectly through the 2 to 3 (3 to 2) link, the decrease in lending to and from 1 results in an observed drop in locational loan flows between 2 and 3 (3 and 2). Taken as ultimate flows, observed locational data would then suggest a decrease in lending from country 2 to 3 (3 to 2), failing to corroborate the model's prediction. This effect may help explain the negative third-country effects documented empirically by Hale, Kapan, and Minoiu (2020), for example.

Another manifestation of the bias is the consistent underestimation of the drop

[29]This interpretation ignores differential intensity of link usage through loops in paths.

[30]We set the persistence of the shock to 0.8.

[31]An alternative would be to calibrate the bilateral intermediation frictions in the comparison economy to exactly match the bilateral network costs.

[32]For example, different collateral liquidation technologies, as in Cao, Minetti, Olivero, and Romanini (2021).

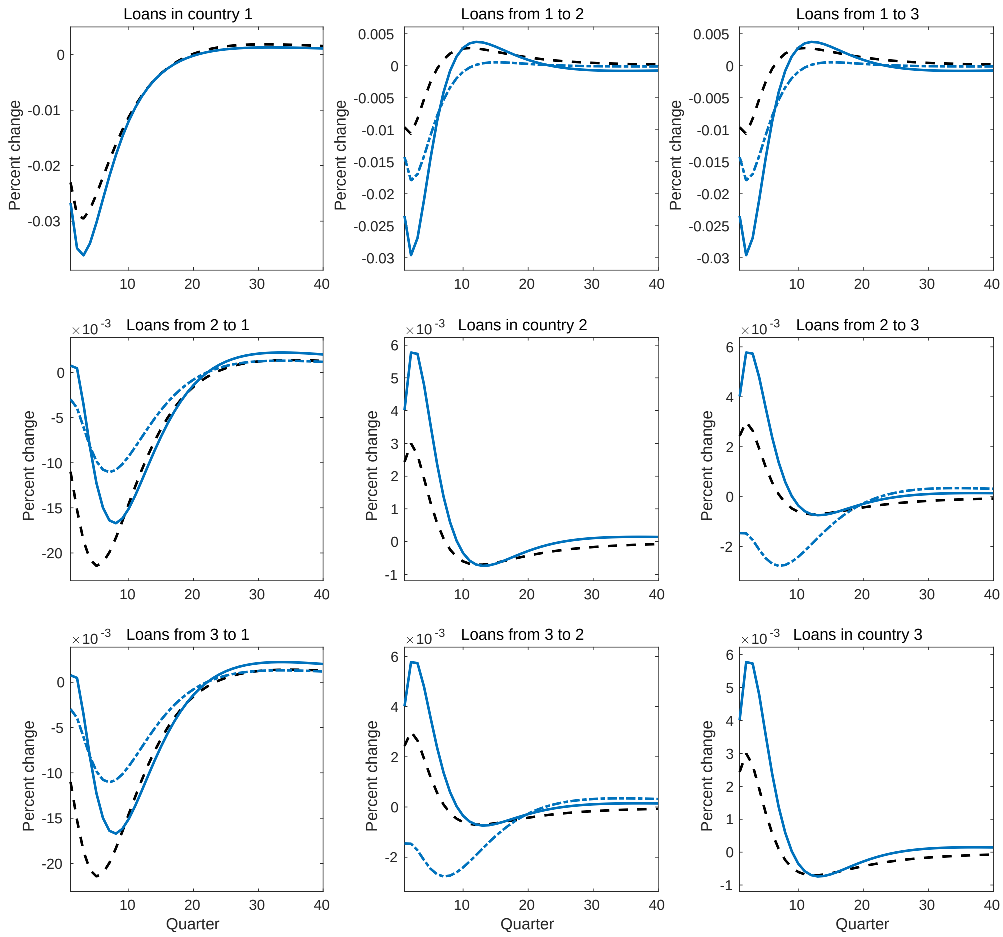


Figure 3: Bilateral Ultimate and Locational Lending, Node Shock

Note: The figure plots the impulse response functions of bilateral loan flows to a 1 standard deviation decrease in the TFP of country 1. The solid blue line refers to ultimate flows in our baseline path economy, the dashed-dotted blue line refers to locational flows, while the dashed black line refer to bilateral lending in an Eaton and Kortum (2002) economy without paths.

in lending to the shocked country 1 when observing locational loan flows. In Figure 3, ultimate loans from 2 to 1 and from 3 to 1 drop substantially, but locational loan flows underestimate the drop. This occurs because some fraction of the substitution (ultimate) lending from country 2 to country 3 and from country 3 to country 2 flows through 1, as captured by the second term in equation (27). The following example helps illustrate this point.

Case study: the 1999 Estonian crisis In 1999, the Estonian real economy suffered a large contraction due to the Russian Rouble Crisis. As a result, domestic bank lending dropped significantly in that year. The major multinational banking conglomerate Nordea, active in Estonia, responded to the crisis by expanding lending to other Nordic and Baltic countries, including Lithuania and Denmark. As part of this strategy, Nordea bank significantly expanded both operations and loan flows to Estonia (doubling flows of funds to Estonia during the period).[33] According to Nordea's reports, this was due to the decision of the banking group to use Estonia as a hub for its expanded activities in other Nordic and Baltic countries.

4.2.2 Edge shocks (regulations or sanctions)

We next consider the case of an edge shock, a counterfactual exercise where bilateral costs are increased (e.g., as a result of regulatory shifts or financial sanctions). While there is a large literature that explores the transmission of shocks that hit a country or a sector, most theoretical works assume an exogenous network. Instead, we allow any bilateral edge to be shocked: recalling Lemma 2, a change in a single e_{ij} has an impact on the entire network cost τ_{ij}.[34]

Proposition 2 (Edge Shock Impact on Loan Flows). *A shock to the edge cost e_{kl} results in the following change in ultimate loan flows:*

$$\frac{\partial X_{ij}}{\partial e_{kl}} = \underbrace{\frac{\partial X_j}{\partial e_{kl}}\lambda_{ij} + X_j \frac{\partial c_i^{-\theta}}{\partial e_{kl}} \frac{\lambda_{ij}}{c_i^{-\theta}} + X_j \frac{\partial R_j^{-\theta}}{\partial e_{kl}} \frac{\lambda_{ij}}{R_j^{-\theta}}}_{\text{GE effects}} + \underbrace{X_j \frac{\partial \tau_{ij}^{-\theta}}{\partial e_{kl}} \frac{\lambda_{ij}}{\tau_{ij}^{-\theta}}}_{\text{network effects}} \tag{28}$$

and the following change in locational flows:

$$\frac{\partial \Xi_{k',l'}}{\partial e_{kl}} = \underbrace{\sum_{i,j} \frac{\partial X_{ij}}{\partial e_{kl}}}_{\text{average effect}} \psi_{k'l'|ij} + \underbrace{\sum_{i,j} X_{ij} \frac{\partial \psi_{k'l'|ij}}{\partial e_{kl}}}_{\text{diversion}}. \tag{29}$$

Equation (28) separates the effect of an edge shock at kl on the ultimate loan flows between i and j into four terms. The first term is the effect of a change in intermediation frictions on total loan demand X_j. The second and third terms capture the effects through the marginal loan production cost and interest rates. The final term captures the changes through bilateral network costs. In an off-the-shelf gravity model, the bilateral network cost in the last term would be replaced with (direct) bilateral frictions, and be zero unless $i = k$, $j = l$. This would bias the first three terms, with the direction of bias depending on the shape of the network.

[33] See, e.g, the 1999 and 2000 annual reports of MeritaNordbanken Group.

[34] Computationally, this entails inverting the B matrix, a $\mathcal{N} \times \mathcal{N}$ matrix of endogenous variables.

Equation (29) separates the effect of an edge shock at kl on the locational loan flows along edge $k'l'$ into two terms: the average effect on ultimate loan flows for all bilateral pairs, weighted by the proportion of each flow using $k'l'$, and the effect on the proportion of ultimate loan flows using $k'l'$, weighted by the size of those ultimate flows. The first term, expanded in equation (28), is the total effect of the edge shock on all lending. The second term represents the extent to which the edge shock diverts loan flows through the network towards or away from $k'l'$.

To illustrate the proposition, we perturb the economy with an unexpected ten percent positive shock to the edge cost $e_{1,2}$.[35] To fix ideas, we can think of country 1 as representing the EU bloc, country 2 Russia, and country 3 other BRICS countries. The shock could capture financial sanctions imposed on lending from the EU to Russia or an increase in regulatory restrictions on the operations of EU banks in Russia. As would be expected, the loan flows X_{12} decrease (see Appendix C for the full set of impulse responses). Figure 4 focuses on the effects between countries 1 (EU) and 3 (other BRICS), between which there is no shock.[36]

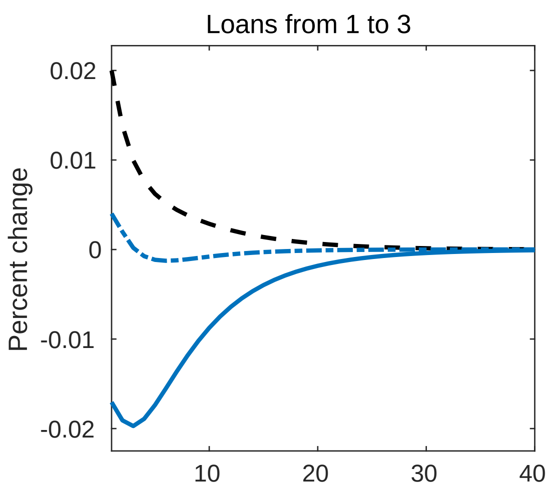


Figure 4: Ultimate and Locational Loans from 1 to 3, Edge Shock

Note: The figure plots plot the impulse response functions of cross-border loans from country 1 to country 2 to a 10% increase in the edge cost e_{12}. The solid blue line refers to ultimate flows in our baseline path economy, the dashed-dotted blue line refers to locational flows, while the dashed black line refer to flows in a standard gravity model à la Eaton and Kortum (2002).

Ultimate loans from the EU bloc to BRICS countries drop too (solid blue line), as the shock also impedes indirect lending between the two through Russia, raising

[35] We set the persistence of the shock to 0.8.

[36] Appendix Figure A1 plots all 9 bilateral flows. Appendix Figure A2 plots bilateral lending flows for permanent edge shocks.

the average cost of bilateral lending. The comparison gravity model, though, cannot account for indirect lending and predicts an increase in lending due to the diversion of country 1 loan flows from 2 (Russia) to 3 (BRICS) countries (dashed black line). In a twist, locational loan flows from 1 to 3 (dashed-dotted blue line) confirm the comparison model with no indirect lending for the wrong reasons. While naive observers would attribute these observed flows to ultimate loan flows, their rise is due to EU banks' funding increasingly reaching Russia through BRICS countries post shock, and not to lending to BRICS countries as substitution.

Edge shocks in the data and the case of the Polish bank deregulation Policy and regulatory shocks that affect global banking groups' ability to operate across countries can be understood through the lens of our model as edge shocks. To identify this kind of regulatory-driven edge shocks, we considered four waves of the database "Banking Regulation and Supervision around the World" compiled by Barth, Caprio and Levine (2020) through questionnaires sent to the banking regulatory and supervisory authorities of 180 countries.[37] The database covers the years from 1996 to 2012. Although there are many cases of regulatory changes that impact national and multinational lending simultaneously, we focus on isolating those instances in which regulations on the activities of multinational banks were altered (temporarily or permanently) without a corresponding change in the regulations of domestic banks—a pure edge shock.[38] We uncovered 30 cases of such regulation-driven edge shocks over the 1996-2012 period, with six episodes constituting very large regulatory shifts (i.e., a lifting or introduction of at least two regulatory measures on global banks).

One such episode revealed by the data consists of the major deregulation of global bank activities in Poland in the second part of the 1990s and the 2000s. Following this deregulation, the Italian multinational banking group UniCredit significantly expanded its activities and lending flows to Poland, while simultaneously slimming down its direct lending and activities in neighboring countries, such as Ukraine. Newly established subsidiaries in Poland were increasingly used to maintain the flow of credit to Ukraine.[39] Observing locational loan flows without consideration of indi-

[37]The coverage is more limited for the first waves of the survey.

[38]Appendix B.6 details the data and our computations.

[39]For a detailed discussion, see, e.g., the 2011-2014 annual reports of UniCredit Group. See also "UniCredit eyes Poland's BGZ, might leave Ukraine: CEO", Reuters, 2013.

rect lending would have led to the false conclusion that apparent loan retrenchments to Ukraine were a sign of lending diversion from Ukraine to Poland, while indirect lending to Ukraine through Poland effectively maintained the ultimate loan flows from Italy to Ukraine.

4.2.3 The response of real sector variables

In Appendix Figure A3 and in Figure 5 we display the impulse responses of salient real sector variables (investment, consumption, labor employed in the final goods sector, and output) to the one percent negative TFP shock in country 1 (Appendix Figure A3) and to the ten percent positive $e_{1,2}$ edge shock (Figure 5). The solid blue lines refer to our economy while the dashed black lines refer to the comparison economy without indirect lending.

As expected, the IRFs in Appendix Figure A3 show a drop in investment in country 1 following the negative TFP shock, accompanied by some investment increase in countries 2 and 3 (due to a lending flight). The IRFs in Figure 5 convey a more insightful message. The $e_{1,2}$ edge shock depresses investment in country 2, now more costly for global banks to reach, while raising investment in country 1, as country 1 banks substitute towards domestic lending.[40] These effects are attenuated in our economy relative to the comparison economy without indirect lending. Put differently, indirect lending appears to mitigate the investment responses in the shock-affected countries. This aggregate stabilizing role of indirect lending also emerges when observing other real sector variables: consumption, labor and output exhibit a smoother response in our setting relative to the comparison setting without indirect lending.

In what follows, we rationalize these observations by examining how indirect lending affects the transmission and propagation of shocks through the banking network.

5 Banking Complexity

In this section, we explore how banking complexity (the prevalence of indirect lending) affects shock propagation. Banking complexity is a function of the dispersion of

[40]Country 3 also exhibits some investment drop, as its banks tend to redirect their credit toward country 2, in partial compensation of the reduced 1 to 2 loan flows.

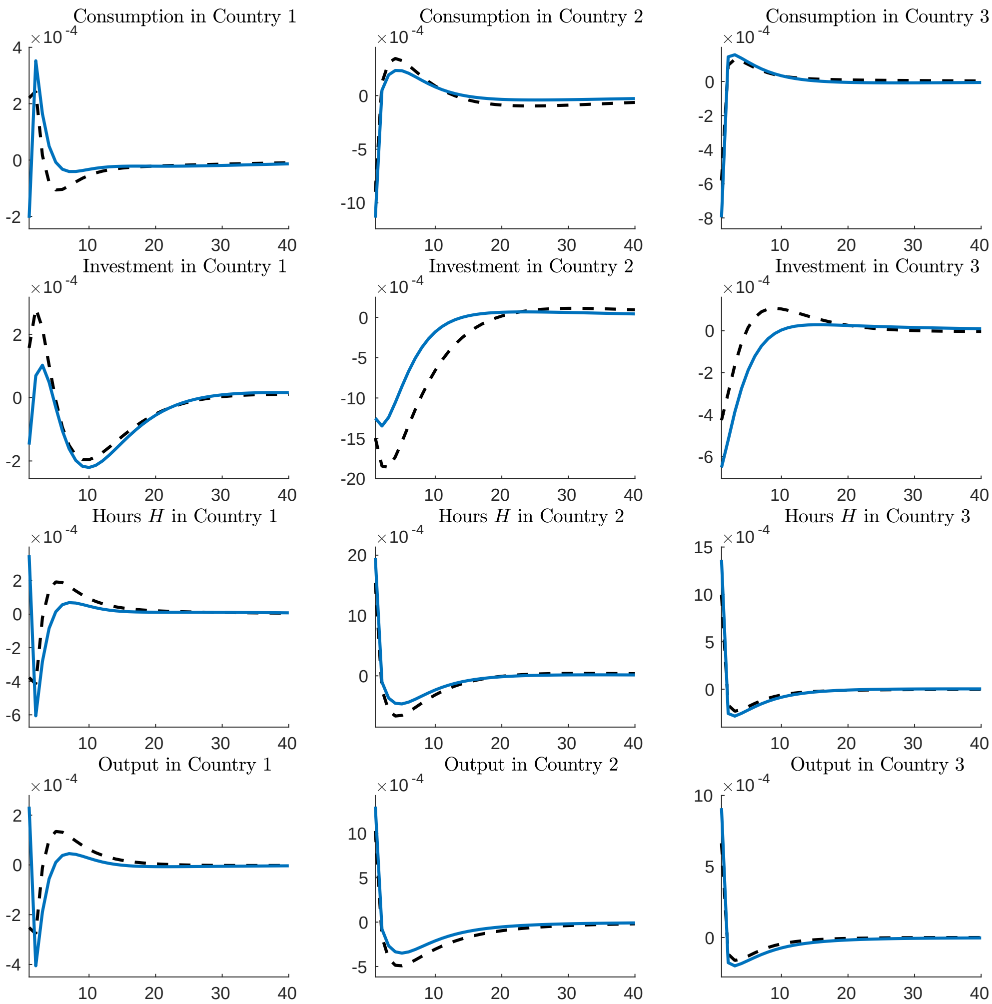


Figure 5: Responses of Real Sector Variables to Edge Shock

Note: The figure plots the impulse response functions of selected real sector variables to a 10% increase in the edge cost e_{12}. The solid blue lines refer to our baseline economy, while the dashed black line refer to an economy without paths, i.e. a standard gravity model à la Eaton and Kortum (2002).

productivity of multinational banking activities across banks. When productivities of particular agent combinations (loan officer or branch) are heterogeneous, the set of cost-minimizing combinations, and thus loan paths, are different across banking groups. Regulatory and technology changes can affect this dispersion. For example, innovations in information communication technology affecting synergies and information sharing between loan officers, regulations affecting the training and mobility of human capital between countries, or regulations governing mergers with local banks (or affecting the degree of restrictions on global syndicated loan networks) could all make specific sets of loan officers or branches, and hence particular lending paths, more efficient for particular banks or loans.

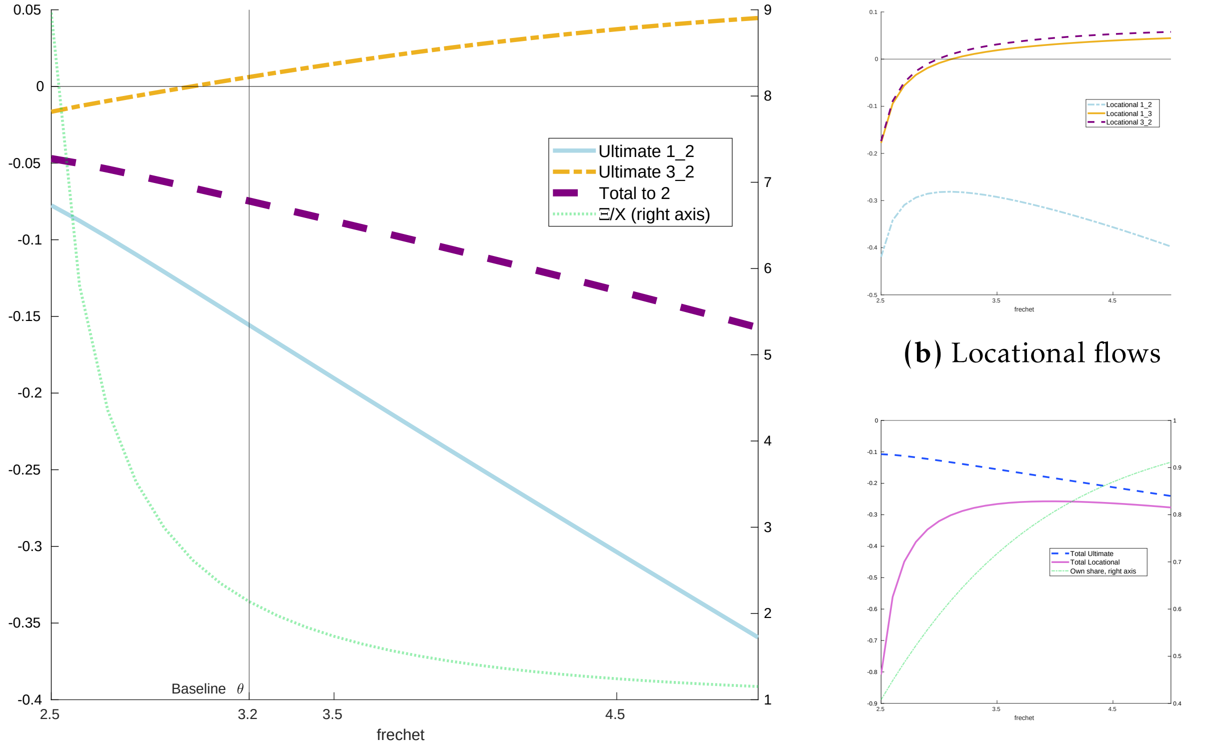
In our model, this dispersion is determined by θ, the parameter governing the relative distribution of idiosyncratic draws and, hence, the prevalence of indirect lending. Lower values of θ imply a thicker tail of idiosyncratic draws, corresponding to a world where ramified banking networks are more active. Policies inducing greater dispersion reduce θ. To investigate the effect of increased dispersion, we therefore repeat the edge shock exercise considered in the previous section while gradually changing the value of θ.

Greater banking complexity (more indirect lending) generates two competing forces. On the one hand, it allows for greater substitution of lending paths: bank lending can circumvent a shocked link between two countries, which mitigates the impact and cross-border transmission of the shock. On the other hand, banking complexity can lead to a lower substitutability of funding sources, as more lending relationships across more countries are exposed to the shock through the network. Importantly, as we will see, here too the mere observation of location loan flows would lead to biased conclusions about the relative strengths of these two competing forces.

We subsequently explore tweaks to our initial experiment, allowing for heterogeneity in the banking efficiency parameter z_i, or simultaneously shocking multiple edges at once. Finally, to further understand the consequences of global banking complexity, in Section 5.3 we will explore financial transformations occurring along an alternative dimension: reductions in the edge costs capturing general reductions in cross-border banking frictions.

5.1 Source and Path Diversification

Lending to country 2 In Figure 6a, we graph the immediate impact of the ten percent positive shock to the edge cost $e_{1,2}$ on ultimate lending flows to country 2 using values of θ ranging from 2.5, the lower value possible to satisfy the spectral radius constraint, to 5, at which point nearly all locational flows are ultimate lending and the domestic lending share increases to above 90%. In Appendix Figure A4, we show robustness to considering the average response over the first four periods after the shock. The solid blue line in Figure 6a represents $X_{1,2}$, ultimate lending from 1 to 2, with 2 being more costly to reach following the edge shock. The dashed-dotted orange line represents $X_{3,2}$, ultimate lending from 3 to 2, with country 3 representing coun-



(a) Ultimate Flows

(b) Locational flows

(c) Total cross-border flows

Figure 6: Banking Complexity and the Response of Loan Flows to Edge Shocks
Note: The figure plots selected variables (y axis) in a scenario with a 10% shock to edge cost e_{12}, at different values of the Fréchet parameter θ (x axis). Panels 6a and plots the impact response of ultimate flows from country 1 to country 2 (solid blue), country 3 to country 2 (dash-dotted orange), and total cross-border flows to country 2 (dashed purple line). The fourth line represents the ratio of global locational to ultimate loan flows (dotted green). Panel 6b plots location flows from country 1 to country 2 (dot-dash blue), country 3 to country 2 (dash purple) and country 1 to 3 (solid orange). Panel 6c plots total ultimate (dashed blue) and locational (solid purple) cross-border flows and the domestic share of ultimate flows (dotted green).

try 2's alternative source of credit. The dashed purple line represents total ultimate lending to country 2. The light green line represents the ratio of global locational to ultimate loan flows.

Consider first ultimate lending from country 1 to country 2 (solid blue line). Unsurprisingly, for any value of θ, loan flows shrink following the shock. Yet, this drop is less pronounced for lower values of the Fréchet parameter, that is, greater banking complexity (i.e., when the global banking network features larger indirect loan flows). To gain insights into this effect, we open up the last term in equation (28). Substituting for the effect on 1 to 2 lending, the network term becomes

$$\underbrace{X_2 \frac{\partial \tau_{12}^{-\theta}}{\partial e_{12}} \frac{\lambda_{12}}{\tau_{12}^{-\theta}}}_{\text{network effects}} = -X_{12}\left[\underbrace{\frac{\theta}{e_{12}} + \frac{\theta}{\tau_{11}}\frac{d\tau_{11}}{de_{12}} + \frac{\theta}{\tau_{22}}\frac{d\tau_{22}}{de_{12}}}_{\text{direct effect}} + \underbrace{\frac{1}{\psi_{12|12}}\frac{d\psi_{12|12}}{de_{12}}}_{\text{switching effect}}\right]. \tag{30}$$

Path diversification has two components: an ex-ante reduction in exposure to a shock, and the ability to divert lending ex post. The first three terms capture the direct effect of the shock on the network-average cost of lending from 1 to 2.[41] Lower values of θ reduce the proportion of loans traveling through the shocked link, diversifying the initial (pre-shock) set of paths and reducing country 1's exposure to the heightened cost of the direct link. The fourth term captures the change in the probability that a given unit of loans uses a path with the direct link. The banking network will give banks in country 1 the ability to shift loans to paths which avoid the direct link, as captured by the fourth term. Lower values of θ increase such ability of banks to switch paths. Together these effects constitute path diversification.

If an interconnected world with large indirect loan flows can benefit from a higher diversification of lending paths, it can, however, lose along an alternative dimension: *source diversification*. Consider ultimate lending from country 3 to country 2 (dashed-dotted orange line in Figure 6a). At relatively high values of the Fréchet parameter (i.e., in a world with little indirect lending), such loan flow responds positively to the $e_{1,2}$ edge shock, reflecting a source-diversification mechanism: increased interest rates in country 2 induce country 3 to act as an alternative source of credit, supplanting the drain of liquidity from country 1. However, in a financially complex economy with more indirect lending (i.e., low θ), this substitution effect is weakened. From Equation (28), the network effect on ultimate lending from 3 to 1 is:

$$X_2 \frac{\partial \tau_{32}^{-\theta}}{\partial e_{12}} \frac{\lambda_{32}}{\tau_{32}^{-\theta}}. \tag{31}$$

The exposure of lending from country 3 to country 2 to the edge shock is proportional to the extent to which country 3 uses country 1 as a hub for lending to country 2 ($\frac{\partial \tau_{32}^{-\theta}}{\partial e_{12}}$), which is decreasing in θ. That is, as θ decreases, we see more loans flowing (initially) indirectly, including from country 3 to country 2, and therefore more use of country 1 as a hub for such transactions. This in turn leaves these substitution lending flows more exposed to the very shock which generates (through general equilibrium effects) their demand. As in Equation (30), a path substitution effect exists for this network effect too, and some portion of these exposed lending flows can divert back to a direct path. However on net, the network effect must be negative (since $\tau_{32}^{-\theta}$ is decreasing in e_{12}) undermining the ability of country 3 to act as an alternative source

[41] The second and third terms account for loans that move internally or circuitously before being lent at 2.

of funding. Indeed, in Figure 6a, as we move left from $\theta = 5$ to $\theta = 2.5$ along the graph, ultimate lending from 3 to 2 shrinks, rather than increases, following the edge shock. From our baseline calibration, we predict this negative effect to begin with further integration. Moreover, the size of the network effect is proportional to λ_{32}, so we should expect this negative network effect to be more important for third countries with bigger, more productive banking systems. Of course, in a model without indirect lending the edge shock between 1 and 2 has no effect on 3 to 2 costs, and there is no loss of source diversification.

The overall effect of the shock on the ultimate loan flows to country 2 (dashed purple line in Figure 6a) depends on how the impact of path diversification compares with that of source diversification. For lower values of the Fréchet parameter, that is, when the global banking network features thicker indirect lending paths, the overall loan flow to country 2 drops less, suggesting that the strengthening of loan path diversification outweighs the weakening of loan source diversification.[42] Below, however, we will see that this conclusion depends on the nature of the shock and on the relative efficiency of the banking sectors of the individual countries. In particular, the relative strengths of the above mechanisms can be reversed when considering scenarios where substitute lending sources are relatively more efficient in loan production.

In Section 4, we found that the conventional approach of looking at locational loan flows can lead to distorted conclusions about the mechanisms of shock transmission. Similarly, observed locational flows can distort conclusions about the consequences of banking complexity. The hump-shaped curve of locational flows from 1 to 2 (blue dashed-dotted line) in Figure 6b appears to show that substantially increasing banking complexity relative to the current calibration of θ amplifies the impact of shocks on loans from 1 to 2, contrary to the actual effect.

Global loan flows Figure 6c shows that the effect of the $e_{1,2}$ edge shock on the global ultimate loan flows is negative, but less so for lower values of θ (higher banking complexity). This reflects the interaction of the aforementioned countervailing forces, and the overall prevalence of a stronger path diversification over the weakening in source diversification when indirect lending is more pervasive (i.e., θ is lower).

Belying the dampening effect of complexification on the impact of shocks to ulti-

[42] In Appendix Figure A5 we show the robustness of these conclusions to considering a setting with $\mathcal{N} = 5$ countries.

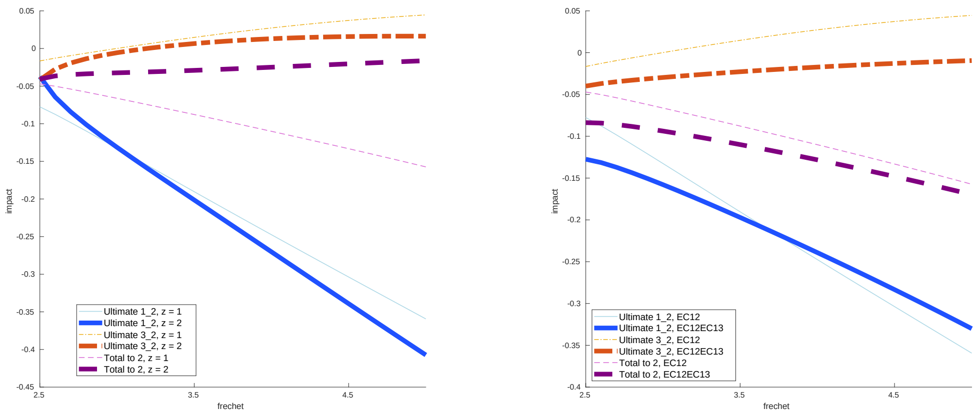
mate lending, locational flows (solid purple line in Figure 6c) once again incorrectly suggest a different story. Higher banking complexity amplifies the negative effect of shocks on locational flows, a product of a larger base share of indirect flows (Figure 6a, green dotted line). Indeed from our baseline calibration of $\theta = 3.2$, observers of locational loan flows would fail to correctly identify the stabilizing effect of banking complexity, perceiving the proliferation of indirect lending as amplifying.

5.2 Can Banking Complexity be Destabilizing?

In this section, we further explore the race between the path and source diversification forces. We first modify our calibration then the nature of the shocks in order to see if the conclusion in the previous section on the consequences of banking complexity could be reversed.

Heterogeneous banking efficiency Countries can differ in the size and efficiency of their banking sectors. Such heterogeneity can shift the relative strengths of the two competing forces, path and loan source diversification. Here, we depart from our baseline symmetric calibration and consider banking complexity in the presence of asymmetries in banking efficiency. To this end, we allow for cross-country differences in loan officers' productivity in producing loans, z_i. If third countries unaffected by a shock feature a highly productive banking sector, their diminished ability to act as alternative sources of liquidity can be particularly harmful and even outweigh the benefits of loan path diversification.

We replicate the previous section's exercise, altering country 3's bank loan officers to be twice as productive (i.e., higher z_i than in countries 1 and 2). Figure 7a overlays the previous results (thin lines) with these new ones (thick lines). The effects of the shock on the loan flows from 1 to 2 and from 3 to 2 are qualitatively similar to the baseline scenario. Indeed, the parameter of the Fréchet distribution, and hence the degree of indirectness of the banking flows, exerts a similar influence on the response to the shock of the ultimate loan flows from 1 to 2 and from 3 to 2. What is instead sharply different from the baseline is the way path diversification and source diversification are weighted against each other. When country 3 is highly efficient in producing loans, losing it as a source of credit exerts a large depressing impact on the total ultimate loans to country 2. While country 1 continues to be able to exploit



(a) Heterogeneous Banking Efficiencies

(b) Multiple Edge Shocks

Figure 7: Banking Complexity and Loan Responses, Alternative Scenarios

Note: Both panels show the impact response (y axis) of selected ultimate loan flows to a 10% shock to edge cost e_{12}, at different values of the Fréchet parameter θ (x axis). The orange dashed-dotted lines refer to flows from country 3 to country 2, the blue solid lines refer to flows from country 1 to country 2, the purple dashed lines to total ultimate cross-border flows to country 2 (i.e. from both 1 and 3). Lighter lines refer to the baseline case (see figure 6a). In Panel 7a, darker lines refer to the corresponding flows in the case of double monitoring efficiency in country 3. In Panel 7b darker lines refer to a scenario with an additional 10% shock to edge cost e_{13}.

country 3 as a tertiary node towards country 2, this path-diversification effect is now dominated by the reduced ability of country 3 to act as an alternative source of credit.

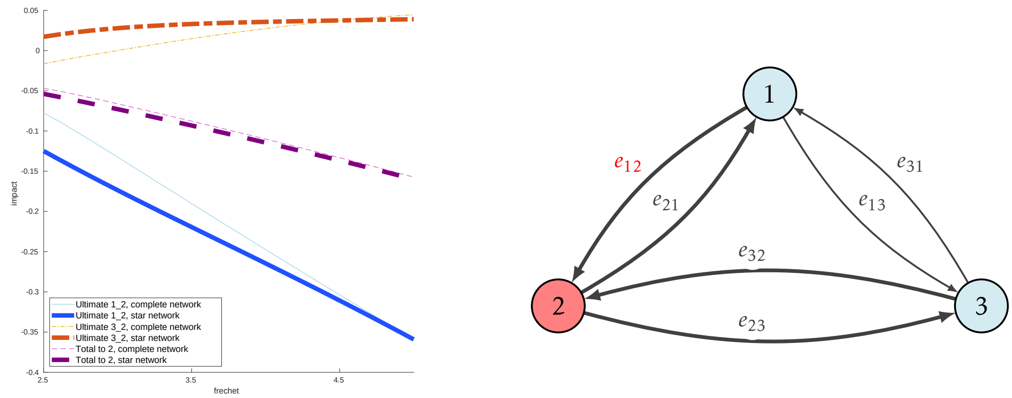
As a result, banking complexity now leads to shock amplification: a global banking network with more indirect linkages (lower θ) exhibits a larger drop in the ultimate loan flows to country 2, as shown by the upward-sloping dashed purple line. This suggests that banking complexity can have a stabilizing influence for shocks between countries with similar degree of banking efficiency but can have a destabilizing effect where countries have significantly different banking efficiency.

Multiple edge shocks In Figure 7b we consider a shock that raises the costs of lending from country 1 to both countries 2 and 3.[43] We again plot these new results in thick lines with baseline results in corresponding thin lines. This double-edge shock can be thought as a tightening of banking regulation in country 1 that increases the cost of lending abroad for banks in 1. The negatively sloped solid black line (the change in the ultimate loans from 1 to 2 under increasing values of θ) is now flatter. Path diversification is now reduced. Due to the increased costs of lending along the 1 to 3 edge, banks in country 1 find it more difficult to circumvent the higher costs of lending directly to 2. On the other hand, increasing complexity still reduces source diversification (observe the positively sloped orange line). While the nature

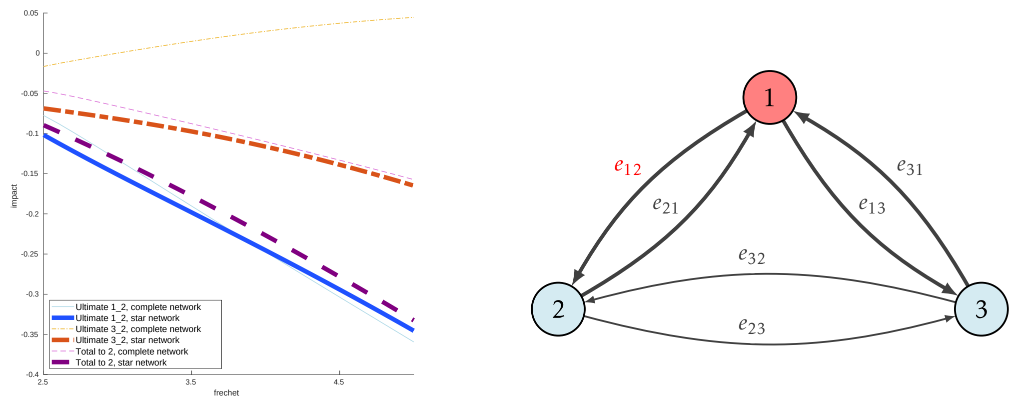
[43] We now increase both $e_{1,2}$ and $e_{1,3}$ by 10%. Again, we set the shock persistence to 0.8.

of the shock reduces path diversification, the qualitative implications of the baseline scenario carry through.

5.3 An Alternative Characterization of Banking Integration



(a) Central node: 2



(b) Central node: 1

Figure 8: Financial Integration via Lower Bilateral Frictions

Note: Both panels show the impact response (y axis) of selected ultimate loan flows to a 10% shock to edge cost e_{12}, at different values of the Fréchet parameter θ (x axis). The orange dashed-dotted lines refer to flows from country 3 to country 2, the blue solid lines refer to flows from country 1 to country 2, the purple dashed lines to total ultimate cross-border flows to country 2 (i.e. from both 1 and 3). Lighter lines refer to the baseline case (see figure 6a). In Panel 8a, darker lines refer to a scenario where country 2 is central, while in Panel 8b country 1 is central.

The reader may wonder whether the effects of banking complexity we uncovered are observationally equivalent to the effects of banking integration via the reduction in cross-border lending frictions (edge costs). Here, we show this is not the case.

We revisit the $e_{1,2}$ edge shock exercise by allowing for different initial levels of integration (bilateral edge costs), then reduce edge costs to obtain a more integrated network (symmetric edge costs). We again re-plot the baseline graph in thin lines under the new results. Since in the exercise we move from counterfactual edge costs back to the calibrated values, we read the graph by moving from thick to thin lines, holding θ fixed.

We consider two scenarios. In a first scenario, we start with poor integration between countries 1 and 3.[44] Integrating 1 and 3 (i.e. reducing the value of $e_{1,3}$) achieves better path diversification following the $e_{1,2}$ shock, as country 1 can now circumvent the higher edge cost $e_{1,2}$ through country 3. In Figure 8a, the thick solid blue line (1 to 2 lending under various θ values before integration) jumps up to the thin solid blue when 3 is integrated. However, the same integration depresses source diversification: the thick dashed orange line drops to the thin dashed yellow line. Intuitively, country 3 was not initially able to exploit indirect lending, and never exposed to the edge shock, but now integrated, its lending to 2 suffers from the $e_{1,2}$ shock. On net, the two effects offset each other, and integration has a negligible influence on the response of total ultimate lending to country 2 following the shock.

In the second scenario, we start with poor integration between countries 3 and 2, and reduce the value of $e_{3,2}$ until we achieve the calibrated edge values. In this case, both source and path diversification strengthen: in Figure 8b integrating 3 pushes all thick lines up to the thin lines. Intuitively, the lack of integration between 3 and 2 inhibited the use of 3 as both an indirect link between 1 and 2 and an alternative source of funding for 2. After integration, both obstacles are removed. In general, integrating two spokes is unambiguously stabilizing for shocks originating from hubs, but ambiguous and potentially destabilizing for shocks originating at the spokes.

6 Endogenous Monitoring

In the main version of the model we assume banks' productivities z_i to be exogenous. However, banks' efficiency in loan production and extension may be affected by experience accumulated through the activity of liquidity intermediation across countries.[45] In this section, we endogenize loan officers' productivity, scaling it by an exponential factor corresponding to the sum of excess inward and outward locational

[44] The relative baseline edge costs (e_{13} and e_{31}) are multiplied by a factor of 2.

[45] In Appendix A, we explicitly model intermediation as a loan officer's task. Alternatively, scale could affect global banking if loan officers' experience in loan production and extension impacts their intermediation efficiency, which would further endogenize the process of hub formation. This is explored in the second part of Appendix A.

loan flows with respect to the country's steady state value:

$$z_{i,t} = z_i * \exp\left[1 + \psi\left(\tilde{\Xi}^{\mathrm{IN}}_{i,t-1} + \tilde{\Xi}^{\mathrm{OUT}}_{i,t-1}\right)\right], \tag{32}$$

In (32), z_i is the steady-state loan officers' productivity of the baseline economy, $\tilde{\Xi}$ are deviations of inward and outward locational flows from their respective steady-state values, and ψ is a positive parameter. This reduced-form approach can capture a learning process where loan officers acquire information and experience based on the number of transactions they process. To capture the persistence of the centrality, we posit that the scale affects the productivity of banks with a lag. Figure 9 plots the loan re

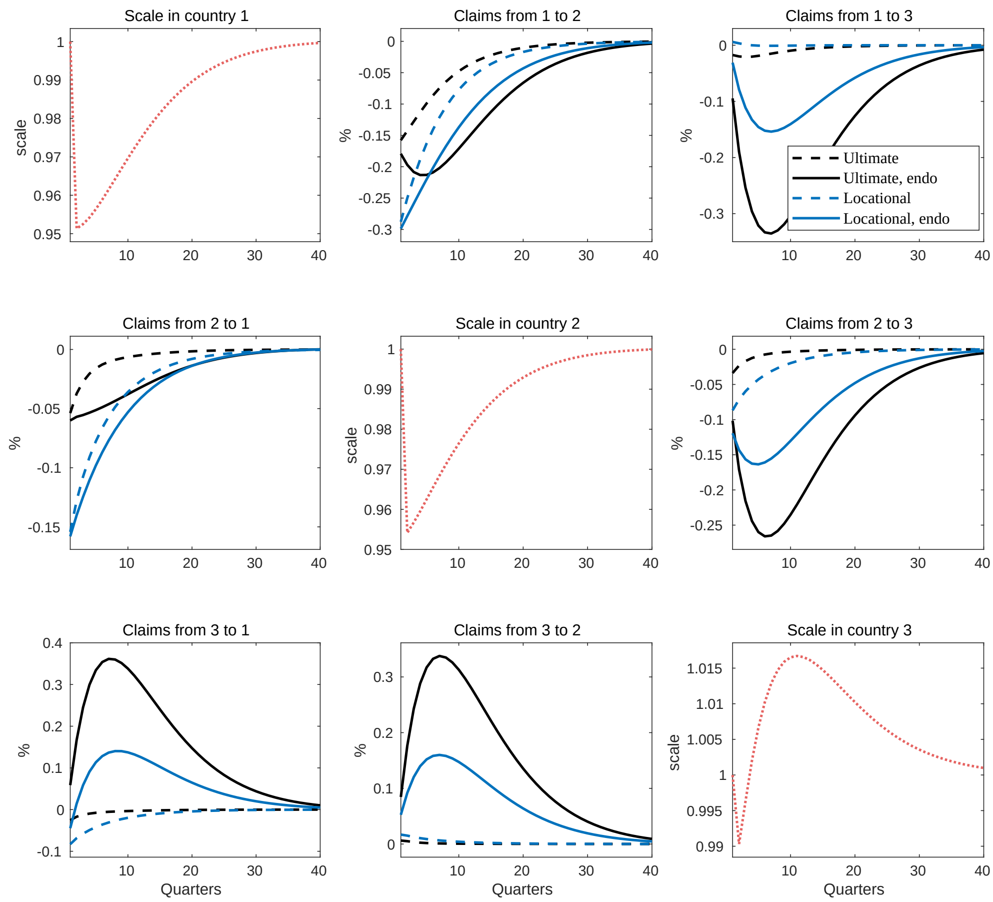


Figure 9: Complexity, Endogenous Banking Productivity

Note: The figure plots the impulse response functions of cross-border loans from country 1 to country 2 to a 10% increase in the edge cost e_{12}. The dashed blue lines refer to ultimate (black) and locational (blue) flows in our baseline path economy, while the solid lines refer to the model with endogenous banking productivity. The dotted red lines on the diagonal figures plot the endogenous scale of the banking productivity for the respective country.

The response of the ultimate loan flows between countries 1 and 2 (solid black line) qualitatively mirrors that obtained in the baseline scenario with exogenous bank productivity (dashed black line). However, the drop in the endogenous banks' productivity triggered by the reduction in loan intermediation shifts that response downward. More surprisingly, the ultimate loan flows from country 3 to countries 1 and 2 increase following the shock. Intuitively, because country 3 becomes an intermediation hub for loan flows from 1 to 2 (path diversification), it rebounds to gain banking efficiency on net, and hence produces more loans, benefiting countries 1 and 2. Interestingly, this suggests that when learning effects are very strong in the banking sector, path diversification could enhance third countries' ability to serve as alternative sources of funds. That is, path and source diversification could exhibit some degree of complementarity.

7 Conclusion

The expansion of complex multinational banking groups with broad networks of affiliates and correspondents is reshaping the global financial landscape. This paper presents a dynamic general equilibrium model with multi-country banking flows to account for the substantial fraction of international lending that is intermediated through banking hubs and complex multi-national routing. Our model rationalizes observable international statistics. It generates a set of bilateral *locational* flows of funds that conceptually matches aggregate (BIS LBS) statistics, as distinct from the *ultimate* demand and supply of bank credit. We show how empirical specifications conflating the BIS LBS data with ultimate origin-destination lending biases results and can distort or even reverse empirical patterns.

We further demonstrate that accounting for indirect banking linkages is crucial for understanding how shocks to international frictions (e.g., bank regulation changes or the introduction of financial sanctions) propagate through the network. Accounting for indirect banking links unveils new tradeoffs when considering banking complexity. While a more complex banking network characterized by thicker indirect links eases the use of alternative paths for reaching destination countries (path diversification), it can undermine countries' ability to diversify the sources of international liquidity (source diversification). Overall, the model yields nuanced implications for macroprudential and regulatory policies: the overall net effect of banking complexity on the propagation of shocks depends on the nature of the shocks and on the relative efficiency of the banking systems of the different countries.

The analysis leaves relevant questions open. While our framework captures a rich structure of network paths, it abstracts from complex general equilibrium interactions between banking intermediation costs and aggregate variables that may be relevant for international lending flows, such as the price of international collateral and financial assets. We leave this and other issues to future research.

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Online Appendices

Banking Complexity in the Global Economy

Online Appendices

Banking Complexity in the Global Economy

These Online Appendices contain further microfoundations for intermediation costs (Appendix A), derivations for the model and further details on the analysis (Appendix B), and additional figures (Appendix C).

A Further Microfoundations

In Section 2.3, we modeled (deterministic) edge costs as e_{kl}. One interpretation of this cost is, as iceberg costs in the trade literature, an additional friction resulting in higher effective marginal cost of monitoring from origin i to destination j. In this section, we provide a microfoundation of this friction along these lines.

A.1 Bank Intermediation and Edge Costs along the Path

In this microfoundation, the banking sector of each country performs two functions: production of loans and intermediation of liquidity between countries. In line with the literature on multinational banks' internal capital markets, we specify liquidity intermediation as an inflow of transfers from loan-originating countries and an outflow of transfers towards destination countries. This mimics the case of the internal capital markets of multinational banking groups which use their own subsidiaries in third countries to reach final destination countries.

The problem of a representative bank is now augmented in two ways. On the constraints side, the bank faces N intermediation constraints (one for each destination country) which relate transfers inwards to transfers outwards, according to a 1:1 technology (i.e., each unit of transfers outwards must be matched by a unit of transfers inwards). This N-constraints specification embeds the idea that banks cannot divert funds committed for specific destinations to cheaper destinations (in order to receive higher edge cost payments and expend lower edge cost payments to such cheaper destinations). On the objective function side, the bank pays costs for operating the liquidity intermediation technology. These can comprise costs for monitoring and managing transfers in and out and for matching them with each other, costs for dealing with other subsidiaries of the banking conglomerate (including informational frictions), loan officer training efforts, and more broadly frictions for using internal capital markets. The bank also receives payments from loan-originating countries aimed at covering such expenses sustained in the intermediation process.

Formally the problem of a bank in node (country) country k reads:

$$\max_{M_k(\omega),D_k(\omega)} \sum_j \frac{r_{kj}(\omega)}{\tau_{kj}(\omega,p)} x_{kj}(\omega) - w_k M_k(\omega) - R_k^D D_k(\omega) - \sum_{j=1}^{N}\sum_{i=1}^{N} e_{ik}\tilde{\Xi}_{ik}^{j} r_{ij}(\omega) + \sum_{i=1}^{N} T_i$$

$$s.t. \quad \sum_j x_{kj}(\omega) = \min\{z_k M_k(\omega), D_k(\omega)\} \tag{A.1}$$

$$\sum_{i=1}^{N} \tilde{\Xi}_{ik}^{j} = \tilde{\Xi}_{kj} \qquad j = 1, \ldots, N, \tag{A.2}$$

where $\tilde{\Xi}_{ik}^{j}$ denotes the transfers into country k from country i which are destined to country j; $\tilde{\Xi}_{kj}$ are the overall transfers from country k to country j; $e_{ik}\tilde{\Xi}_{ik}^{j} r_{ij}$ represents the edge costs sustained by the bank for intermediating the transfers $\tilde{\Xi}_{ik}^{j}$; and T_i is the total payment received from each origin country i for the purpose of covering the intermediation expenses sustained by this bank in k. Two observations are in order about the intermediation costs. First, in this specification the banks in k must sustain intermediation costs for managing transfers inwards (the analysis would be similar if costs had to be sustained for managing transfers outwards). Second, as implied by the main text analysis, the payments T_i received from the loan-originating countries effectively cover all the expenses sustained for intermediation costs.

Finally, note that the problem above rules out that transfers are used in the loan production technology, as this would effectively imply that transfers are seized by the intermediating bank of country k. In a similar vein, allowing for local deposits to be used in the intermediation activity would imply that ξ draws are no longer forced to be associated with origin banks and their specific source of funds.

A.2 Bank Labor and Edge Costs

Below, we lay out a slightly richer microfoundation where edge costs are costs paid to labor for monitoring and managing liquidity transfers at intermediate nodes. For notational simplicity, in what follows we drop the time subscript t.

In addition to labor costs for loan origination, banks choosing route p now pay monitoring costs for loan intermediation. As before, the deterministic elements of intermediation frictions are:

$$\tilde{\tau}_{ij}(p) = \prod_{k=1}^{K_p} e_{k-1,k}.$$

However, now $e_{k-1,k}$ reflects endogenous labor costs for loan intermediation:

$$e_{k-1,k} = \frac{w_k^I}{z_{k-1,k}^I} \tag{A.3}$$

where w_k^I is the wage paid in country k to workers engaged in (monitoring) loan intermediation activities, and z_k^I is the node's bilateral monitoring efficiency in intermediation activities involving transfers from country $k-1$.

The intermediation constraints faced by a bank at node k now become:

$$\sum_{i=1}^{N} \min\left\{z_{i,k}^I m_{ik}^I(\omega), \tilde{\Xi}_{ik}^j(\omega)\right\} = \tilde{\Xi}_{kj} \qquad j = 1, ..., N, \tag{A.4}$$

where m_{ik}^I is the amount of labor employed in monitoring transfers from i to k. Here, bilateral frictions are due to the need of paying for a loan's monitoring in its travel along p. Effectively, loan production is global, and the loan production function is only the domestic portion of loan production.

In a scenario in which labor for intermediation activities and for loan production activities are differentiated from each other, the total demand for labor in intermediation activities reads

$$M_k^I = \sum_i m_{ik}^I = \sum_i \sum_j \frac{\tilde{\Xi}_{ik}^j}{z_{i,k}^I}.$$

The supply of labor for intermediation activities comes from households, whose modified problem reads:

$$\max_{\{C_{k,t},H_{k,t},M_{k,t},M^I_{k,t}\}_{t\geq 0}} \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\ln C^M_{k,t} - k_H \frac{H_{k,t}^{1+\epsilon}}{1+\epsilon} - k_M \frac{M_{k,t}^{1+\varphi}}{1+\varphi} - k_I \frac{{M^I_{k,t}}^{1+\eta}}{1+\eta} \right)$$

$$s.t. \qquad C_{k,t} + D_{k,t} = (1 + R^D_{k,t-1}) D_{k,t-1} + w^H_{k,t} H_{k,t} + w^M_{k,t} M_{k,t} + w^I_{k,t} M^I_{k,t} + \Pi_{k,t}.$$

Thus, households also derive disutility from a third type of labor (for bank intermediation tasks) and receive wages on such labor. In this setup, edge costs are endogenous to banking wages for intermediations tasks, $w^I_{k,t}$. Increased demand for using k as a node can raise banking wages for intermediation activities and affect edge costs, as long as $\eta > 0$. However, in a setting where $\eta = 0$ and households suffer from linear disutility in performing intermediation tasks, the wage rate $w^I_{k,t}$ for intermediation will be fixed (possibly normalized to a value consistent with the data), and the analysis would remain as in the main text.

B Derivations and Further Details

In this appendix, we provide more details on the model's derivations. In the appendix, with some definitional abuse, we use price and interest rate interchangeably.

B.1 Details on Households and Firms

Households' first order conditions read

$$[C_{i,t}]: \quad 1 = \mathbb{E}_t \Lambda_{i,t,t+1} (1 + R^D_{i,t}), \tag{B.1}$$

$$[H_{i,t}]: \quad k_H H^{\epsilon}_{i,t} = \frac{w^H_{i,t}}{C_{i,t}}, \tag{B.2}$$

$$[M_{i,t}]: \quad k_M M^{\varphi}_{i,t} = \frac{w^M_{i,t}}{C_{i,t}}, \tag{B.3}$$

Firms' first order conditions, in turn, read:

$$[H^D_{i,t}]: \quad \frac{(1-\alpha) Y_{i,t}}{H_{i,t}} = w^H_{i,t}, \tag{B.4}$$

$$[K_{i,t}]: \quad -P^K_{i,t}(1 + R^X_{i,t}) + \mathbb{E}_t \left[\Lambda_{i,t,t+1} \left((1-\delta) P^K_{i,t+1} (1 + R^X_{i,t+1}) + \frac{\alpha Y_{i,t+1}}{K_{i,t}} \right) \right] = 0. \tag{B.5}$$

B.2 Proof of Lemma 1 - Gravity Path Probability

Firms receive bids for financing their capital investments. Banks are competitive and each bank from country i and industry ω makes firms face the same interest rate R_i^X. Hence, price is given by:

$$r_{ij}^X(\omega) = c_i^X \ \tau_{ij}(\omega). \tag{B.6}$$

The goal is to derive the probability that a route is the lowest-cost route from i to j for loan product ω and country i is the lowest-cost supplier of loan product ω to j. We want to know the probability that any given loan ω is sent from i to j on a specific route p. Firms choose the lowest-cost route from i to j for ω from all routes $p \in G$ and firms in j choose the lowest-cost supplier of loan ω from all countries $i \in I$. We will observe ω being sent on a route from i to j if the final interest rate of ω including both the marginal cost of loan production and the shipping cost on that route from i to j, $p_{ijnr}(\omega)$, is lower than all the other interest rates of loan ω from all the other country-route combinations.

Therefore, we will find i) the probability that a country i provides loans to country j at the lowest price; ii) the price of the loan that a country i actually pays to country j is independent of j's characteristics.

B.2.1 Lenders

The *unconditional* probability that taking a route p to lend from country i to j for a given loan product ω costs less than a constant τ is:

$$\begin{aligned} H_{ijp\omega}(\tau) \equiv & \Pr\left(\tau_{ij}(p,\omega) \leq \tau\right) \\ =& 1 - \exp\left\{ -\left[\frac{\tilde{\tau}_{ij}(p)}{\tau}\right]^{-\theta} \right\}. \end{aligned} \tag{B.7}$$

Because the technology is i.i.d across types, this probability will be the same for all loan products $\omega \in \Omega$.

So far we have considered the potential intermediation cost. However, we do not observe bilateral ex-ante costs, but the cost that each country applies ex-post, after choosing the cheapest path. The probability that, *conditional* on banks choosing the least cost route, the cost in ω is less than some constant τ is given by:

$$H_{ij\omega}(\tau) \equiv \Pr\left(\tau_{ij}(\omega) \leq \tau\right),$$

which, after some algebra, yields

$$1 - \exp\left\{ - \tau^{\theta} \sum_{p \in G} \left[\tilde{\tau}_{ij}(p) \right]^{-\theta} \right\}. \tag{B.8}$$

To summarize, this is the probability that, given that banks choose the lower cost route, the cost is below a certain value.

B.2.2 Borrowers

The probability that the interest rate is below a certain constant is

$$\begin{aligned} G_{ijp\omega}(r) \equiv & \Pr\left(r_{ij}(p,\omega) \leq r\right) \\ = & 1 - \exp\left\{ - \left[c_i \frac{\tilde{\tau}_{ij}(p)}{r} \right]^{-\theta} \right\}. \end{aligned} \tag{B.9}$$

Firms minimize the interest rate they pay across countries and routes:

$$\begin{aligned} G_{j\omega}(r) \equiv & \Pr\left(\min_{i \in I, p \in G} r_{ij}(p,\omega) \leq r \right) \\ = & 1 - \exp\left\{ - r^{\theta} \sum_{i \in I} c_i^{-\theta} \sum_{p \in G} \left[\tilde{\tau}_{ij}(p) \right]^{-\theta} \right\}. \end{aligned} \tag{B.10}$$

B.2.3 Market Making

Finally, we can combine the two sides of the market, i.e. the probability that a firm in country j chooses to borrow from a bank of country i, and that the route from country i to j is the minimal cost route. In other words, we compute the probability that, picking any other route-country pair, the interest rate will be higher than the optimal one.

$$\pi_{ijp\omega} \equiv \Pr\left(r_{ij}(p,\omega) \leq \min_{k\neq i,\, s\neq p} r_{kj}(s,\omega)\right)$$
$$= \frac{\left[c_i \tilde{\tau}_{ij}(p)\right]^{-\theta}}{\sum_{i'\in I} c_{i'}^{-\theta} \sum_{p\in G}\left[\tilde{\tau}_{i'j}(p)\right]^{-\theta}}. \tag{B.11}$$

By the law of large numbers, given the continuum of loan products, this is also the share of all loans extended from i to j in industry ω and that take route p, $\lambda_{ijp\omega}$.

B.3 Proof of Lemma 2 - Expected Cost

Assume banks choose the route that minimizes the cost of sending a loan from country i to country j. Let A be the inverse cost matrix defined above, and denote by b_{ij} an element of the matrix $B \equiv (\mathbf{I} - A)^{-1}$. Then, the network cost τ_{ij} is:

$$\tau_{ij} = \gamma b_{ij}^{-1/\theta}$$

where

$$b_{ij} = \sum_{p\in G_{ij}} \tilde{\tau}_{ij}(p)^{-\theta} \equiv \sum_{p\in G_{ij}} \prod_{k=1}^{K_p} e_{k-1,k}(p)^{-\theta}.$$

B.3.1 Expected Cost

The cost between locations i and j is the expected intermediation cost τ_{ij} from i to j across all lenders. Using also the distribution in (B.8):

$$\tau_{ij} \equiv \mathbb{E}_\omega\left[\tau_{ij}(p)\right] = \int_{p\in G_{ij}} \tau_{ijp}(\omega)\, dp = \int_0^\infty \tau \, dH_{ij\omega}(\tau)$$
$$= \Gamma\left(\frac{1+\theta}{\theta}\right)\left[\sum_{p\in G_{ij}} \tilde{\tau}_{ijp}^{-\theta}\right]^{-1/\theta}. \tag{B.12}$$

B.3.2 Expected Cost with Paths

Let $\gamma \equiv \Gamma\left(\frac{1+\theta}{\theta}\right)$. Following Allen and Arkolakis (2022) and taking into account the length of the path, and all possible lengths:

$$\tau_{ij}^{-\theta} = \gamma^{-\theta} \sum_{p \in G_{ij}} \left[\tilde{\tau}_{ij}(p)\right]^{-\theta} = \gamma^{-\theta} \sum_{K=0}^{\infty} \sum_{p \in G_{ij}(K)} \left[\tilde{\tau}_{ij}(p)\right]^{-\theta}$$
$$= \gamma^{-\theta} \sum_{K=0}^{\infty} \sum_{p \in G_{ij}(K)} \prod_{k=1}^{K} e_{k-1,k}^{-\theta} = \gamma^{-\theta} \sum_{K=0}^{\infty} A_{ij}^{K},$$

where we have used the definition $e_{k-1,k}^{-\theta} \equiv a_{ij}$. As long as the spectral radius of A is less than one, then:[46]

$$\sum_{K=0}^{\infty} A^K = (\mathbf{I} - A)^{-1} \equiv B. \tag{B.13}$$

Hence:

$$\tau_{ij} = \gamma b_{ij}^{-1/\theta} \Leftrightarrow b_{ij} = \sum_{p \in G_{ij}} \left[\tilde{\tau}_{ij}(p)\right]^{-\theta}. \tag{B.14}$$

B.4 Proof of Lemma 3 - Locational Gravity

The probability of going through an edge kl, conditional on origin i and destination j, is:

$$\psi_{kl|ij} = \sum_{K=0}^{\infty} \sum_{p \in G_{ij}^{kl}(K)} \frac{\tilde{\tau}_{ij}(p)^{-\theta}}{\sum_{K=0}^{\infty} \sum_{p \in G_{ij}(K)} \left[\tilde{\tau}_{ij}(p)\right]^{-\theta}}$$
$$= \frac{1}{b_{ij}} \sum_{K=0}^{\infty} \sum_{p \in G_{ij}^{kl}(K)} \tilde{\tau}_{ij}(p)^{-\theta} = \frac{1}{b_{ij}} \sum_{K=0}^{\infty} \sum_{p \in G_{ij}^{kl}(K)} \prod_{k=1}^{K} e_{k-1,k}(p)^{-\theta}$$
$$= \frac{1}{b_{ij}} \left(b_{ik} a_{kl} b_{lj}\right), \tag{B.15}$$

where in the last step we isolate the kl step and follow the matrix algebra in Allen and Arkolakis (2022), such that $\sum_{K=0} \sum_{L=0}^{K-1} A^L A A^{K-L-1} = (I-A)^{-1} A (I-A)^{-1}$.

[46] A sufficient condition for the spectral radius being less than one is if $\sum_j e_{ij}^{-\theta} < 1$ for all i.

The conditional probability is:

$$\psi_{kl|ij} = \frac{b_{ik}a_{kl}b_{lj}}{b_{ij}} = \left(\gamma \frac{\tau_{ik}e_{kl}\tau_{lj}}{\tau_{ij}}\right)^{-\theta} \tag{B.16}$$

where the last step was obtained by plugging the expected cost definition in (17).

B.5 Aggregate Interest Rate

Let $G_{ij}(\phi)$ be the Pareto (equilibrium) probability density function of the productivities of banks from country i that lend to country j such that the measure of banks from country i with productivity ϕ is $N_i\, dG_i(\phi)$. Then we can write the aggregate interest rate in j as:

$$R_j = \vartheta\left(\sum_i c_i^{-\theta} b_{ij}\right)^{-\frac{1}{\theta}} \tag{B.17}$$

where $\vartheta = \Gamma\left(\frac{\theta+1-\sigma}{\theta}\right)^{\frac{1}{1-\sigma}}$. In fact,

$$\begin{aligned} R_j^{1-\sigma} &= \int_\Omega r_{ij}(\omega)^{1-\sigma}\, d\omega = \int_0^\infty p^{1-\sigma}\, dG_j(p) = \int_0^\infty p^{1-\sigma} \frac{d}{dp}(1-\exp\{-p\Phi\})\, dp \\ &= \Phi^{-\frac{1-\sigma}{\theta}}\Gamma\left(\frac{\theta+1-\sigma}{\theta}\right), \end{aligned}$$

from which (B.17) follows.

B.6 Identification of Edge Shocks in the Data

As noted in Section 4.2.2, to assess the restrictiveness of the regulation for domestic and multinational banks, we considered four waves of the Caprio and Levine (2020) "World Bank Surveys on Bank Regulation" database spanning the 1996-2012 period and covering a maximum of 180 countries. We used as indicator of domestic bank regulation the "entry into banking requirements" index and as indicator of multinational bank regulation the "limitations on foreign bank entry/ownership" index. We then computed the changes in these indicators across survey waves (whenever consecutive data across two survey waves were available). Finally, we identified all instances in which there was an increase or decrease of the index of regulation on multinational banks not accompanied by a corresponding change in the domestic bank regulation index. We counted 30 such episodes. In six of these episodes the index of regulation on multinational banks changed by two units or more, signalling the introduction or removal of at least two restrictions on multinational banks' operations.

C Appendix Figures

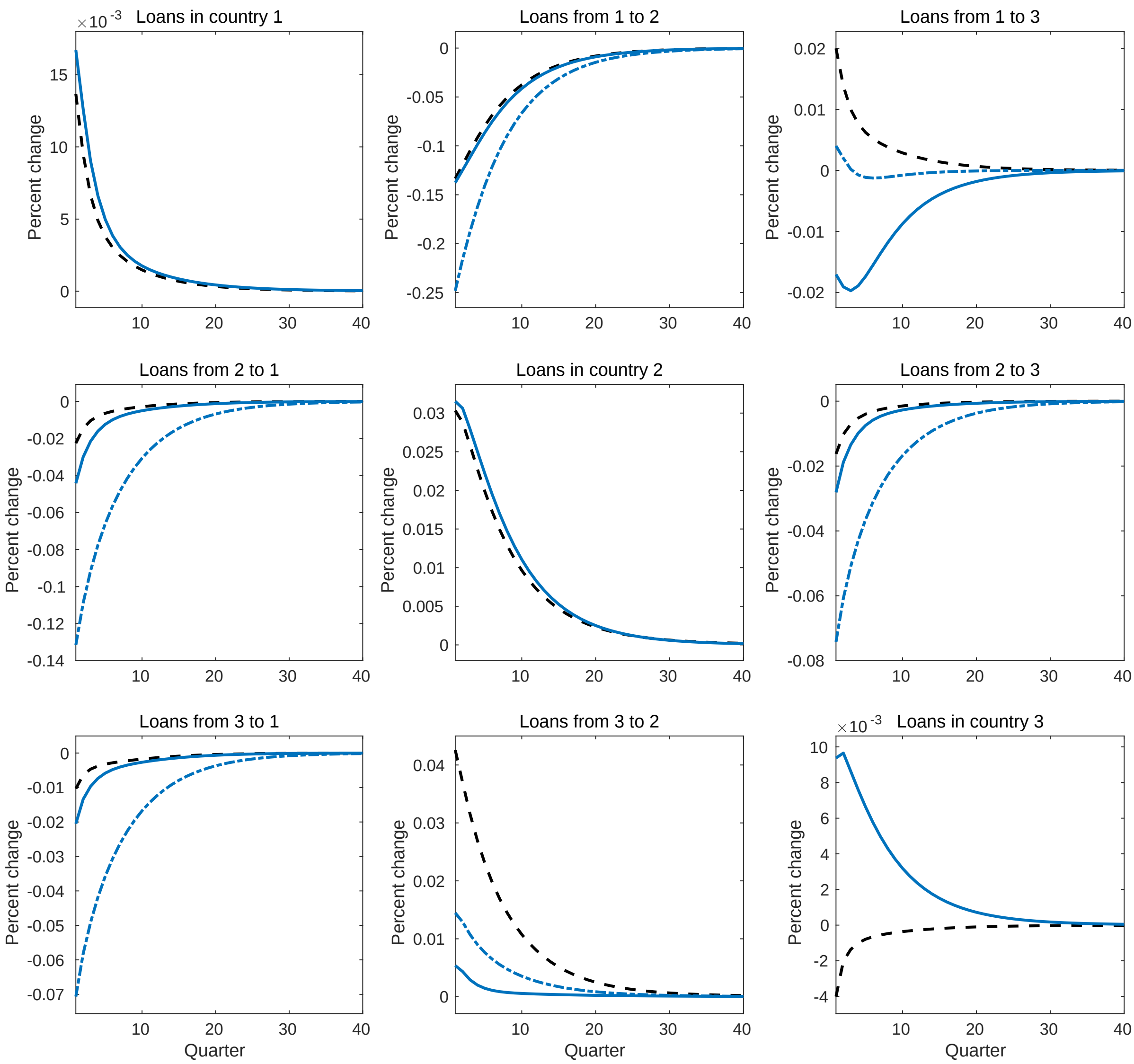

Figure A1: Edge Shock, Full Set of Loan Responses

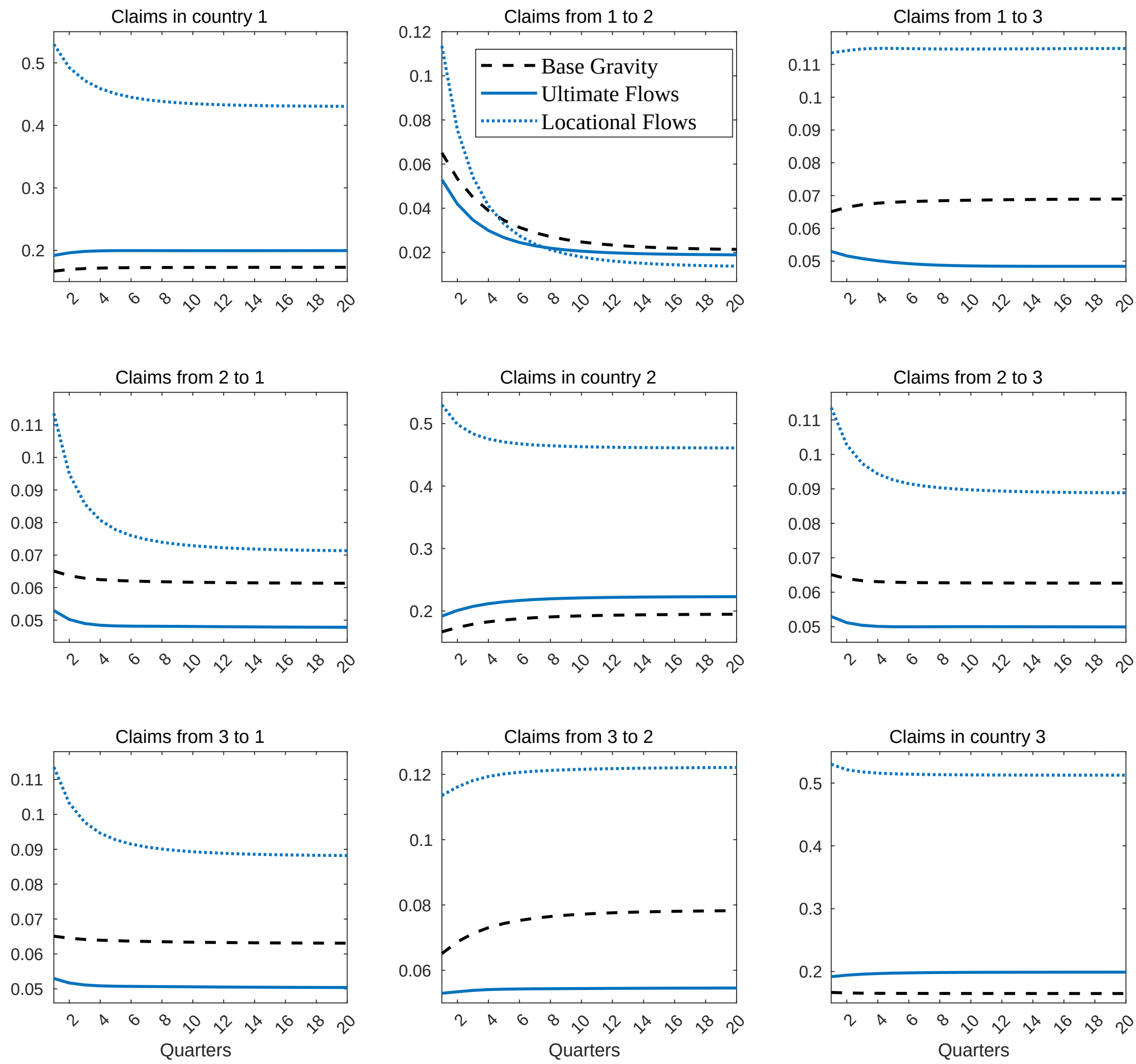

Figure A2: Loan Responses to a Permanent Edge Shock

Note: The figure plots responses to a 200% edge shock to $e_{1,2}$. The persistence of the shock is set to 0.6.

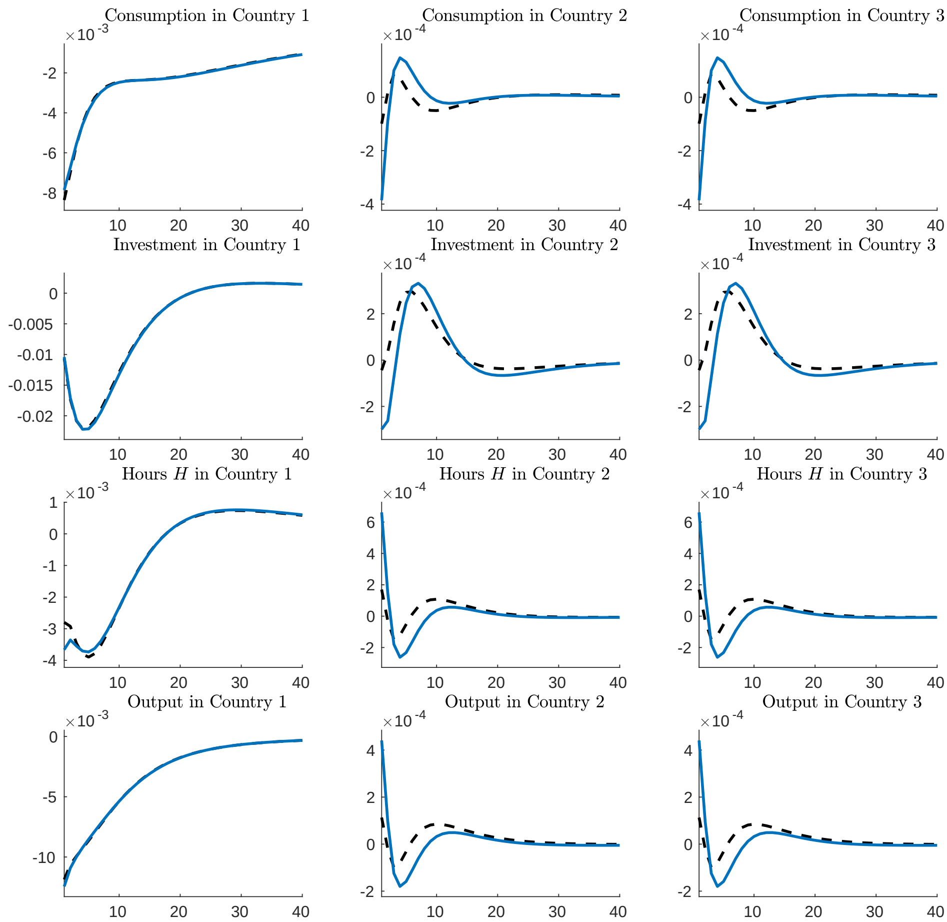

Figure A3: Node Shock, Real Variables Responses

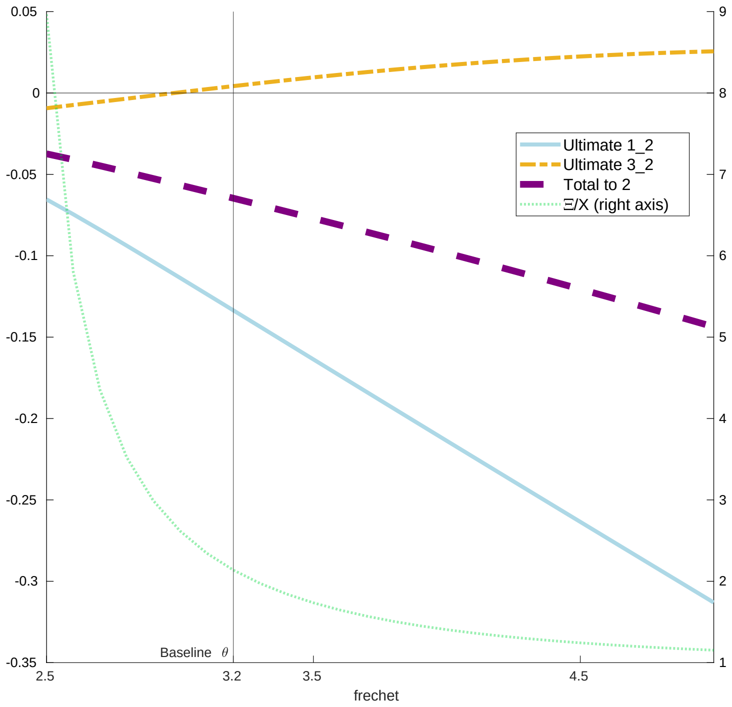

Figure A4: Complexity and Loan Responses to Edge Shocks, 4-Period Average

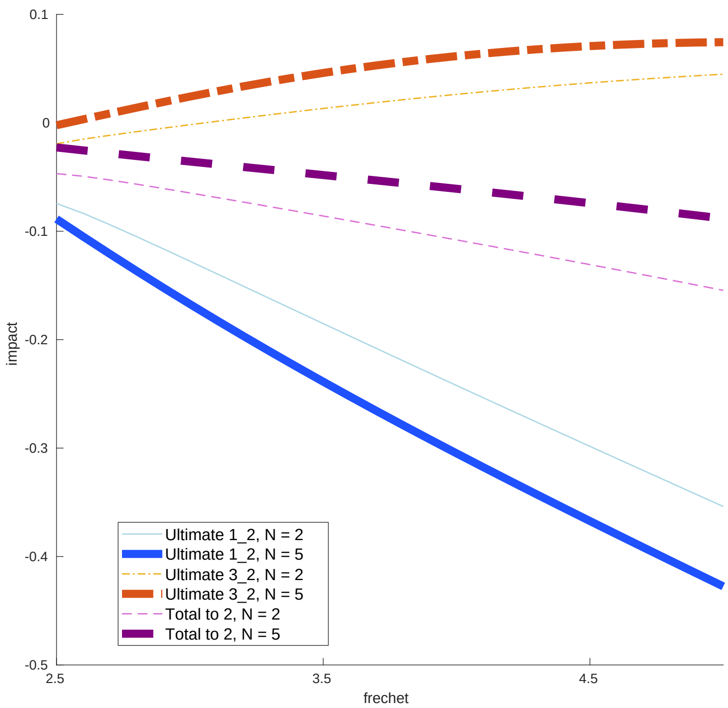

Figure A5: Complexity and Loan Responses to Edge Shocks, 3 and 5 country economies

Note: In the N = 5 case, the response labelled as from country 3 to country 2 represents the sum of country 3,4, and 5 to country 2.